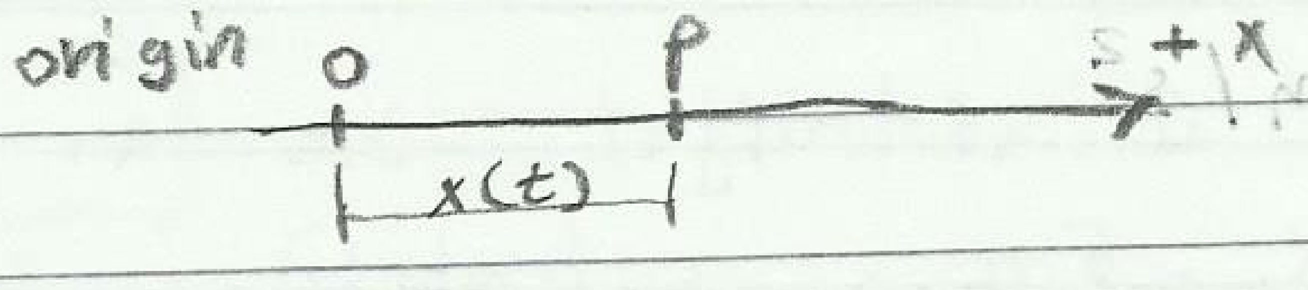
13/02/2012

Dynamics

Kinematics	Kinetics
study the properties of the motion regardless بدون الاخذ في الاعتبار its reasons. (forces)	relate the properties of motion & its reasons.

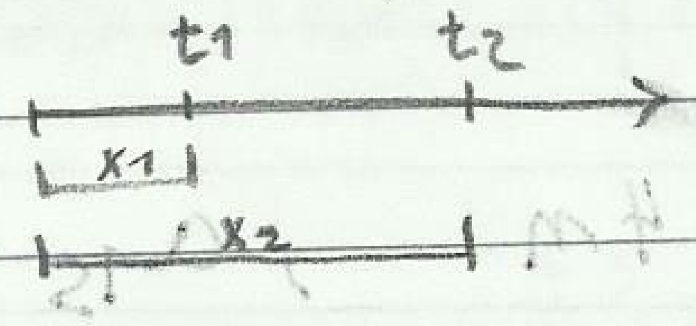
Kinematics of a particle

Rectilinear motion



① position : $x(t)$

② Displacement : الازاحة



$$\boxed{d_{t_1 \to t_2} = \Delta x = x_2 - x_1}$$

③ velocity : v متجه السرعة

$$\boxed{v(t) = \frac{dx}{dt}}$$

④ Acceleration.

$$\boxed{a(t) = \frac{dv}{dt}}$$

Example

A particle moves along a straight line such that

$$x(t) = t^2 - 2t + 5$$

calculate :

1. the initial position, velocity, accelaration.
2. when the velocity is zero ? what's the position & accelaration at this time ?
3. * The position, displacement & total distance traveled at $t = 3s$ [0,3]

solution.

1. $x = t^2 - 2t + 5$ m

$v = \frac{dx}{dt} = 2t - 2$ m/s

$a = \frac{dv}{dt} = 2$ m/s^2

at $t = 0 \Rightarrow x_0 = 5m$, $v_0 = -2 m/s$, $a_0 = 2 m/s^2$

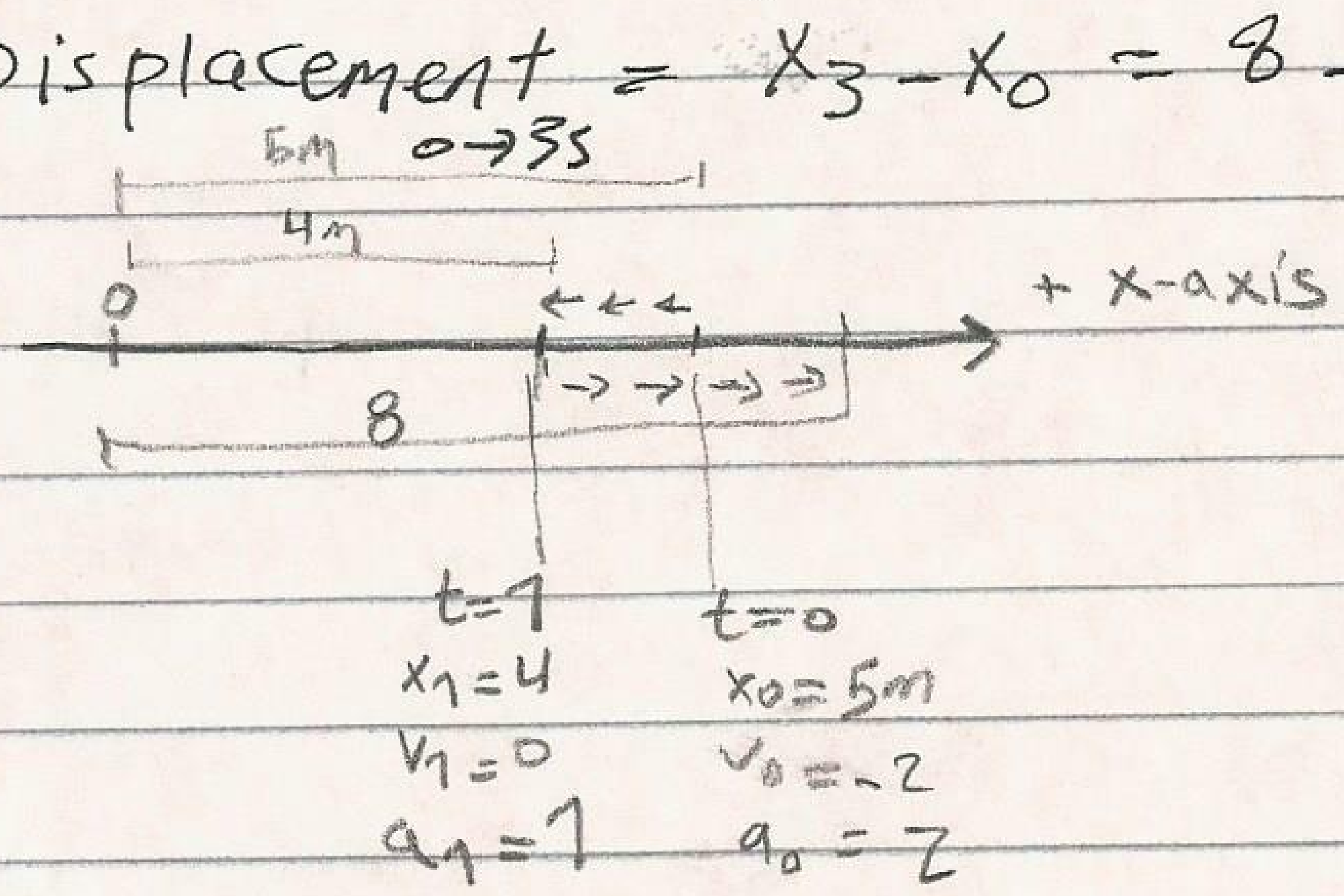
2. let $v = 0 \Rightarrow 2t - 2 = 0 \Rightarrow t = 1s$

(change in direction)

$x_{1s} = 4m$, $a_{1s} = 2 m/s^2$

3. at $t = 3s \Rightarrow x_{3s} = (3)^2 - 2(3) + 5 = 8 m$

Displacement $= x_3 - x_0 = 8 - 5 = 3m$



sign of a   يتحرك شمال او يمين (− / +)

type of a < accelerating rate   نفس الاشارة (++) او (−−)

deccelerating rate   عكس الاشارة (+ −)

total distance traveled

$$= \underset{(\leftarrow)}{1} + \underset{(\rightarrow)}{4} = 5\,m$$

tatal distance traveled

$$= |x_{v=0} - x_{initial}| + |x_{final} - x_{v0}|$$

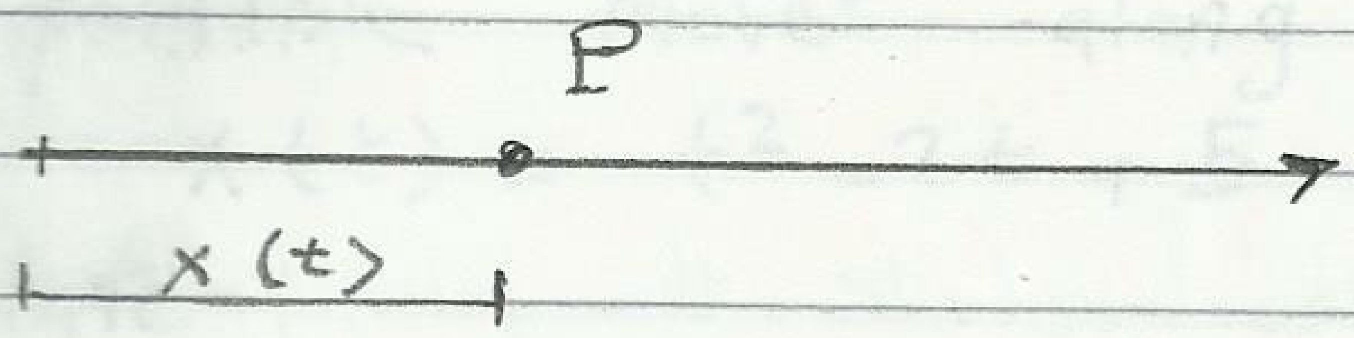
$$= |x_{1s} - x_0| + |x_3 - x_1|$$

$$= |4 - 5| + |8 - 4| = 1 + 4 = 5\,m$$

tut.

25/02/2012

Rectilinear motion



$$x(t) \underset{\int dt}{\overset{\frac{d}{dt}}{\rightleftarrows}} v = \frac{dx}{dt} \underset{\int dt}{\overset{\frac{d}{dt}}{\rightleftarrows}} a = \frac{dv}{dt}$$

Displacement $= x_{final} - x_{initial}$

at $v = 0 \longrightarrow$ the direction must be changed.

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problem (11.14) page 614

apartical in astraight line $\boxed{a = 9 - 3t^2}$

at $t = 0 \longrightarrow v_0 = 0 \ , \ x_0 = 5m$ calculate:

1. The time when the velocity is again zero
2. position & velocity at $t = 4s$.
3. The displacement & total distance travelled at $t = 4s$.

$$a = 9 - 3t^2$$

let $a = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = 9 - 3t^2 \Rightarrow \int dv = \int (9 - 3t^2)\, dt$ بنفصل المتغيرات

$$v = 9t - t^3 + C_1$$

at $t = 0 \longrightarrow v_0 = 0 \Longrightarrow C_1 = 0$

$$\boxed{v = 9t - t^3}$$

OR

$$\int_0^v dv = \int_0^t (9 - 3t^2)\, dt$$

$$|v|_0^v = |9t - t^3|_0^t \Rightarrow v - 0 = (9t - t^3) - 0$$

$$\boxed{v = 9t - t^3}$$

let $v = \frac{dx}{dt}$

$\frac{dx}{dt} = 9t - t^3 \Rightarrow \int_5^x dx = \int_0^t (9t - t^3)\, dt$

$|x|_0^5 = |4.5t^2 - 0.25t^4|_0^t$

$x = 4.5t^2 - 0.25t^4 + 5$

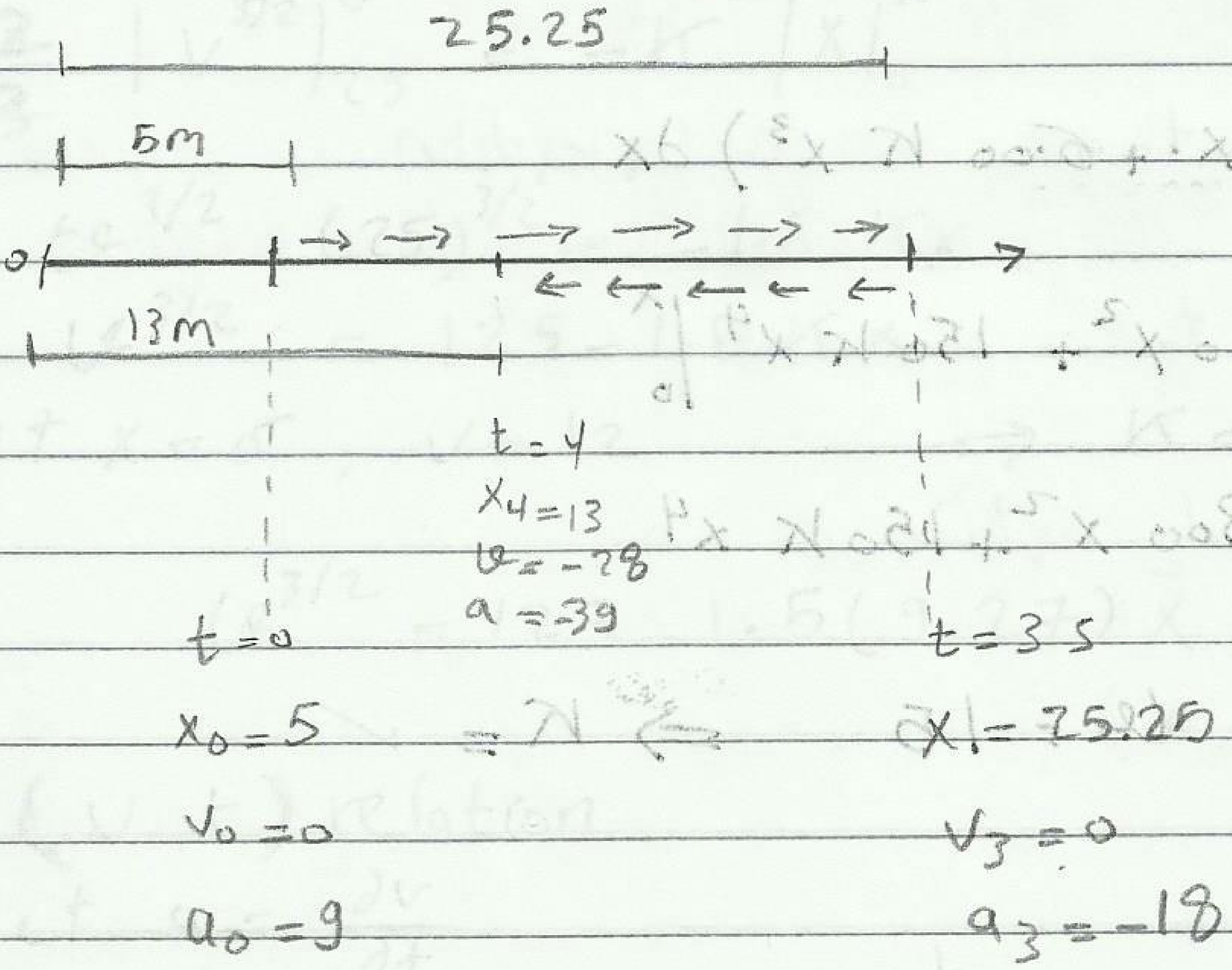
let $v = 0 \longrightarrow 9t - t^3 = 0 \to t = 0,\ 9 - t^2 = 0 \to t = 3,\ \cancel{t = -3}$

at $t = 0 \Rightarrow x_0 = 5$, $v_0 = 0$ $a_0 = 9\ m/s^2$

at $t = 3 \Rightarrow x_3 = 25.25m$, $v_3 = 0$, $a_3 = -18\ m/s^2$

at $t = 4 \Rightarrow x_4 = 13\ m$, $v = -28\ m/s$, $a_4 = -39\ m/s^2$

Displacement $= x_4 - x_0 = 13 - 5 = 8\ m$



total distance $= \overrightarrow{20.25} + \overleftarrow{12.25} = 32.5\ m$

$|x_3 - x_0| + |x_4 - x_3|$

* إذا م يعطيني v_0 ، لازم اجيبها لانه طالب Total distance

problem (11.18) page 614

$$a = 600x + 600\,K\,x^3 \leftarrow \quad K \text{ is a constant}$$

$x = 0 \longrightarrow v_{x=0} = 7.5\ m/s$

$x = 0.45m \longrightarrow v_{x=.45} = 15\ m/s$

calculate : the velocity as a function of position $v(x)$
& the constant K.

$$a = \frac{\partial v}{\partial t}$$

$$a = \left(\frac{dx}{dt}\right)\left(\frac{dv}{dx}\right) = v\,\frac{dv}{dx}$$

let $a = v\,\frac{dv}{dx}$

$$v\,\frac{dv}{dx} = 600x + 600Kx^3$$

$$\int_{7.5}^{v} v\,dv = \int_0^x (600x + 600\,K\,x^3)\,dx$$

$$\left|\frac{v^2}{2}\right|_{7.5}^{v} = \left|300x^2 + 150Kx^4\right|_0^x$$

$$\frac{v^2}{2} - \frac{(7.5)^2}{2} = 300x^2 + 150Kx^4$$

at $x = 0.45m$, $v = 15 \Rightarrow K = \checkmark$

problem (11.25) page 615

$a = -k\sqrt{v}$ $\quad$ k is a constant.

at $t = 0 \Rightarrow x_0 = 0$ ($v_0 = 25$ m/s

$x = 6$ m , $v_{x=6m} = 12$ m/s

calculate: * the velocity at $x = 8$ m.

* the time required to come to rest ($v = 0$)

(v-x) relation.

let $a = v\frac{dv}{dx}$

$$v\frac{dv}{dx} = -k\, v^{\frac{1}{2}}$$

$$\int_{25}^{v}\frac{v}{v^{\frac{1}{2}}}\,dv = -k\int_{0}^{x}dx$$

$$\frac{2}{3}\left|v^{3/2}\right|_{25}^{v} = -k\,|x|_{0}^{x}$$

$$v^{3/2} - (25)^{3/2} = -1.5\,kx$$

$$v^{3/2} = 125 - 1.5\,kx$$

at $x = 6$, $v = 12$ $\quad \Rightarrow k = 9.27$

$$\boxed{v^{3/2} = 125 - 1.5(9.27)\,x}$$

(v-t) relation

let $a = \frac{dv}{dt}$

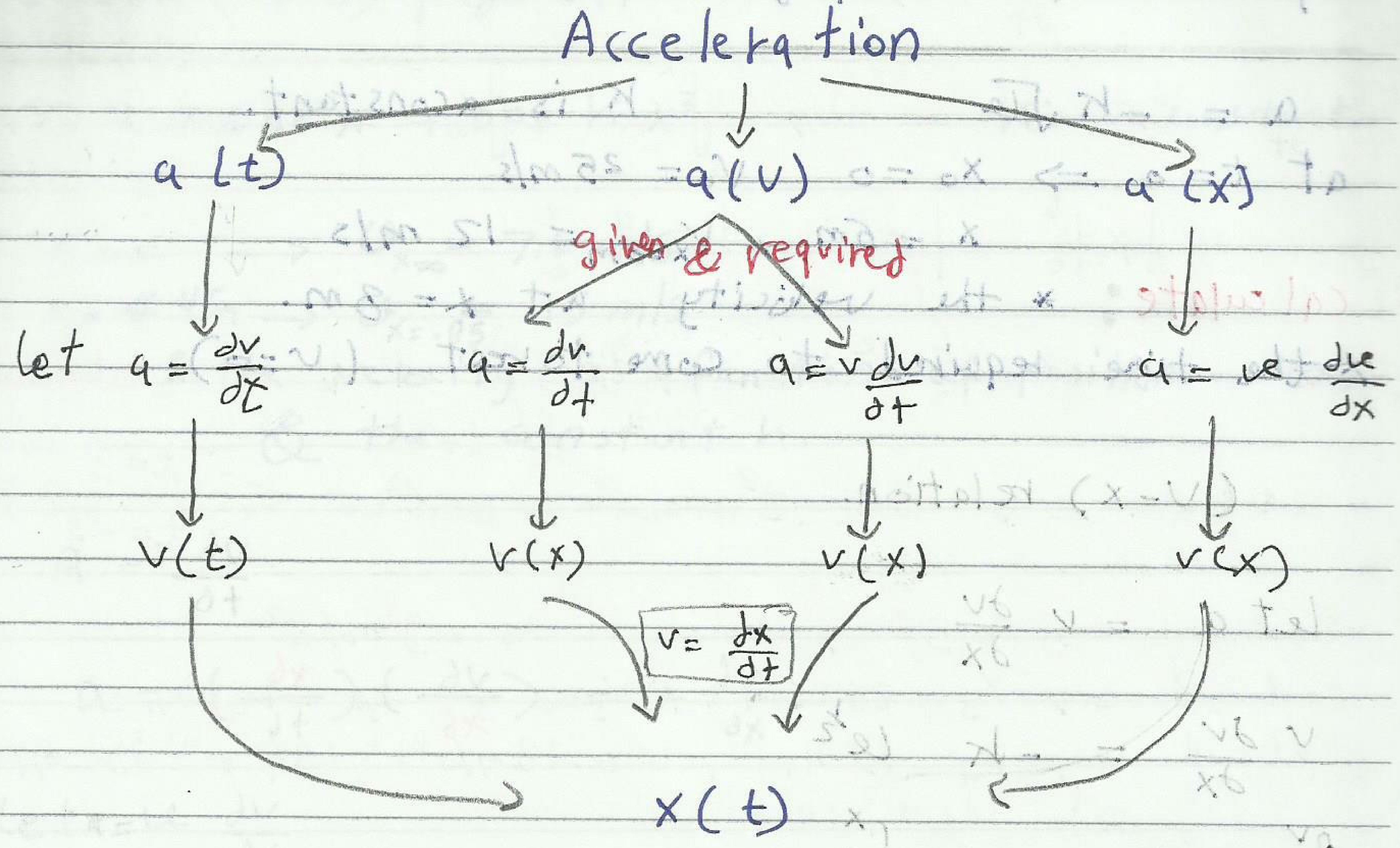
$$\frac{dv}{dt} = -9.27\,(v)^{\frac{1}{2}}$$

$$\int_{25}^{v} v^{-\frac{1}{2}}\,dv = -9.27\int_{0}^{t}dt$$

$$2\left|\sqrt{v}\right|_{25}^{v} = -9.27\,t$$

$$2(\sqrt{v} - 5) = -9.27\,t$$

at $v = 0 \Longrightarrow t = \frac{10}{9.27}$ s



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for <u>constant</u> accelaration.

$$v = v_0 + at$$

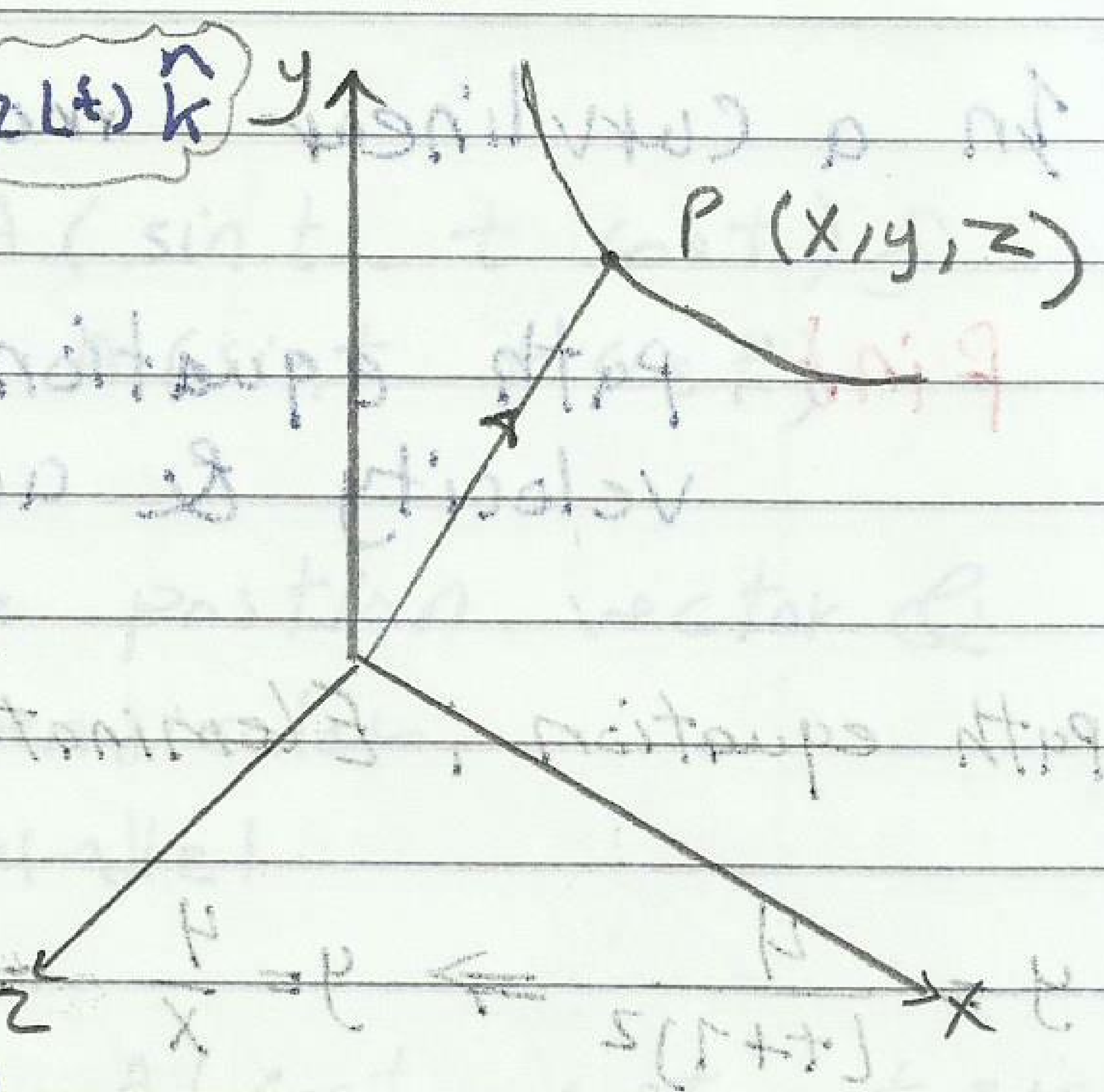
$$d = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2ad$$

Lect. 2

Curvilinear motion

Postition: $\boxed{r(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}}$



Velocity: $\vec{v}(t) = \frac{\partial r}{\partial t}$

$$\vec{v} = \frac{\partial x}{\partial t}\hat{i} + \frac{\partial y}{\partial t}\hat{j} + \frac{\partial z}{\partial t}\hat{k}$$

$$= \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k}$$

speed = magnitude of velocity

$$= \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

Accelration: $\vec{a}(t) = \frac{\partial v}{\partial t}$

$$\boxed{\vec{a} = \frac{\partial^2 x}{\partial t^2}\hat{i} + \frac{\partial^2 y}{\partial t^2}\hat{j} + \frac{\partial^2 z}{\partial t^2}\hat{k} = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{k} + \ddot{z}\,\hat{k}}$$

magnitude of accelration $= \sqrt{\ddot{v}^2 + \ddot{y}^2 + \ddot{z}^2}$

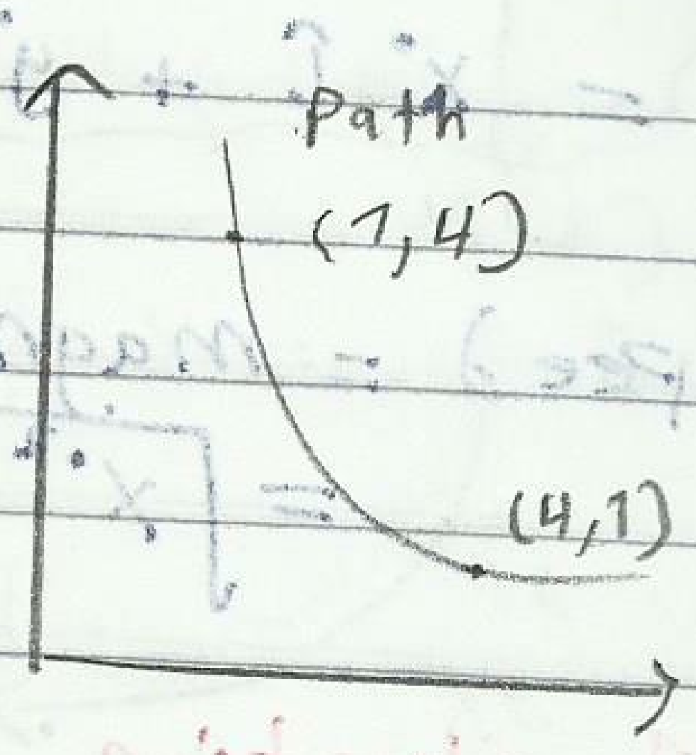
problem (11.91) page 623

In a curvlinear motion: $x = (t+1)^2$, $y = 4(t+1)^{-2}$

find: path equation.
velocity & accelaration at $t=0$

path equation ; Eleminate t betwen x,y

$$y = \frac{4}{(t+1)^2} \Rightarrow y = \frac{4}{x} \Rightarrow \boxed{xy = 4}$$



$x = (t+1)^2$ | $y = 4(t+1)^{-2}$

$\dot{x} = \frac{dx}{dt} = 2(t+1)$ | $\dot{y} = \frac{dy}{dt} = -8(t+1)^{-3}$

$\ddot{x} = \frac{d^2x}{dt^2} = 2$ | $\ddot{y} = \frac{d^2y}{dt^2} = 24(t+1)^{-4}$

at $t=0$

$\dot{x} = 2 m/s$, $\ddot{x} = 2 m/s^2$

$\dot{y} = -8 m/s^2$, $\ddot{x} = 24 m/s^2$

$$\vec{V}_0 = \dot{x}_0 \hat{i} + \dot{y}_0 \hat{j} = 2\hat{i} + 8\hat{j}$$

$$\vec{a}_0 = \ddot{x}_0 \hat{i} + \ddot{y}_0 \hat{j} = 2\hat{i} + 24\hat{j}$$

$$\tan\theta = \frac{y}{x}$$

problem (11.95) page 653

A particle in a curvlinear

$\vec{r} = A(\cos t + t\sin t)\hat{i} + A(\sin t - t\cos t)\hat{j}$

$x = A(\cos t + t\sin t)$ $\qquad y = A(\sin t - t\cos t)$

Calculate the time at which the postion vector & accelration vector are i) perpendicular.

ii) parallel.

$\dot{x} = A(-\sin t + \sin t + t\cos t)$ | $\dot{y} = A(\cos t - \cos t + t\sin t)$

$= At\cos t$ | $= At\sin t$

$\ddot{x} = A(\cos t - t\sin t)$ | $\ddot{y} = A(\sin t + \cos t)$

case 1 $\quad \vec{r} \perp \vec{a} \Rightarrow \vec{r}\cdot\vec{a} = 0$

$x\ddot{x} + y\ddot{y} = 0$

$A^2(\cos t + \sin t)(\cos t - t\sin t) + A^2(\sin t - t\cos t)(\sin t + \cos t) = 0$

$\cancel{A^2}[\cos^2 t - t^2\sin t + \sin^2 t - t^2\cos^2 t] = 0$

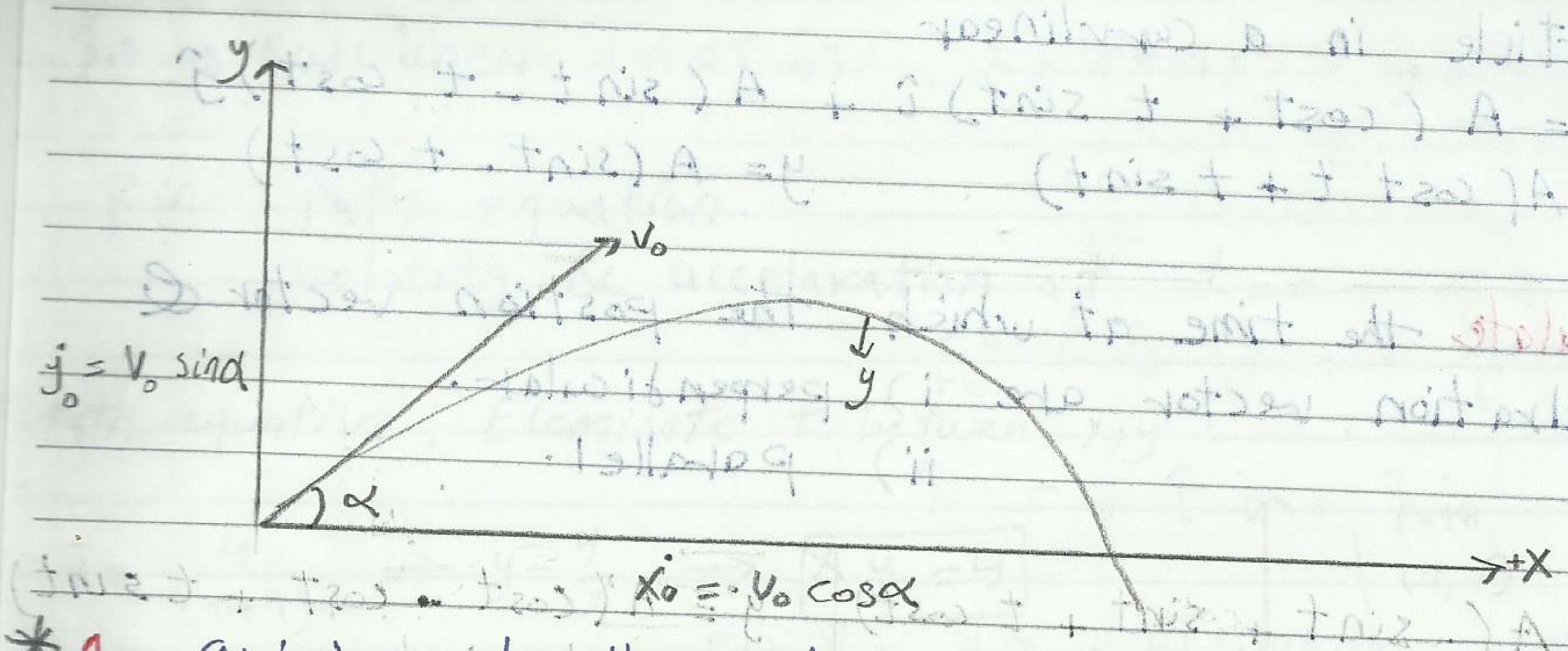
$1 - t^2 = 0 \quad t = 1, \quad \cancel{t = -1}$

case 2 $\quad \vec{r} \parallel \vec{a} \Longrightarrow \vec{r}\times\vec{a} = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0 \\ \ddot{x} & \ddot{y} & 0 \end{vmatrix} = 0$$

$x\ddot{y} - y\ddot{x} = 0 \Longrightarrow t = 0$

Curvlinear motion (projectiles) مقذوفة



* 1- Origin at the intial point.
2- X-axis is the direction of projectile.
3- y-axis is vertically upword.

Accelerations:

$\boxed{a_x = \ddot{x} = 0}$ ① $\boxed{a_y = \ddot{y} = -g}$ ②

$v_x = \dot{x} = c_1$ $v_y = \dot{y} = -gt + c_2$

at $t=0$ $\dot{x}_o = V_o \cos\alpha$ $\dot{y} = V_o \sin\alpha$

$\boxed{v_x = \dot{x} = V_o \cos\alpha}$ ③ $\boxed{v_y = \dot{y} = V_o \sin\alpha - gt}$ ④

$x = (v_o \cos\alpha) t + c_3$ $y = (v_o \sin\alpha) t - \frac{1}{2} g t^2 + c_4$

at $t = 0 \Rightarrow x_o = 0$ $y_o = 0$

$\boxed{x = (V_o \cos\alpha) t}$ ⑤ $\boxed{y = (v_o \sin\alpha) t - \frac{1}{2} g t^2}$ ⑥

$\boxed{\dot{y}^2 = (V_o \sin\alpha)^2 - 2gy}$ ⑦

path equation; هات آخر 5 وعوضها في 6

$$\boxed{y = x \tan\alpha - \frac{g x^2}{2 v^2 \cos^2\alpha}} \quad 8$$

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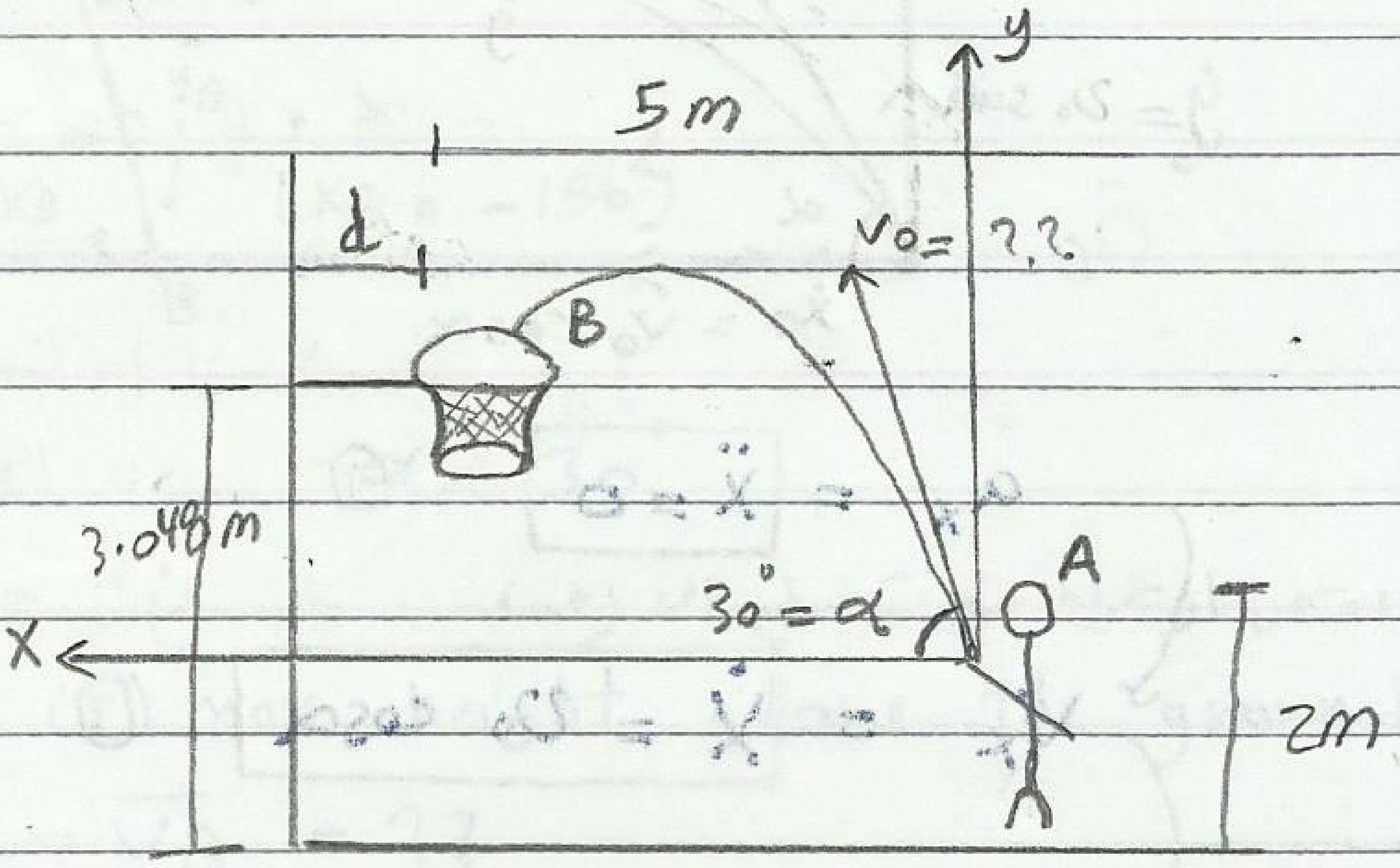
problem (11.110) page 656



Calculate V_0

case 1 $d = 228$ mm

case 2 $d = 430$ mm.

Calculate $t_{A\to B}$

case 1 $d = 228$ mm $= 0.228$ m

at B $x_B = 5 - d$ $\quad y_B = 3.048 - 2$ $\quad \alpha = 30°$

$$y = x \tan\alpha - \frac{g x^2}{2 V^2 \cos^2\alpha}$$

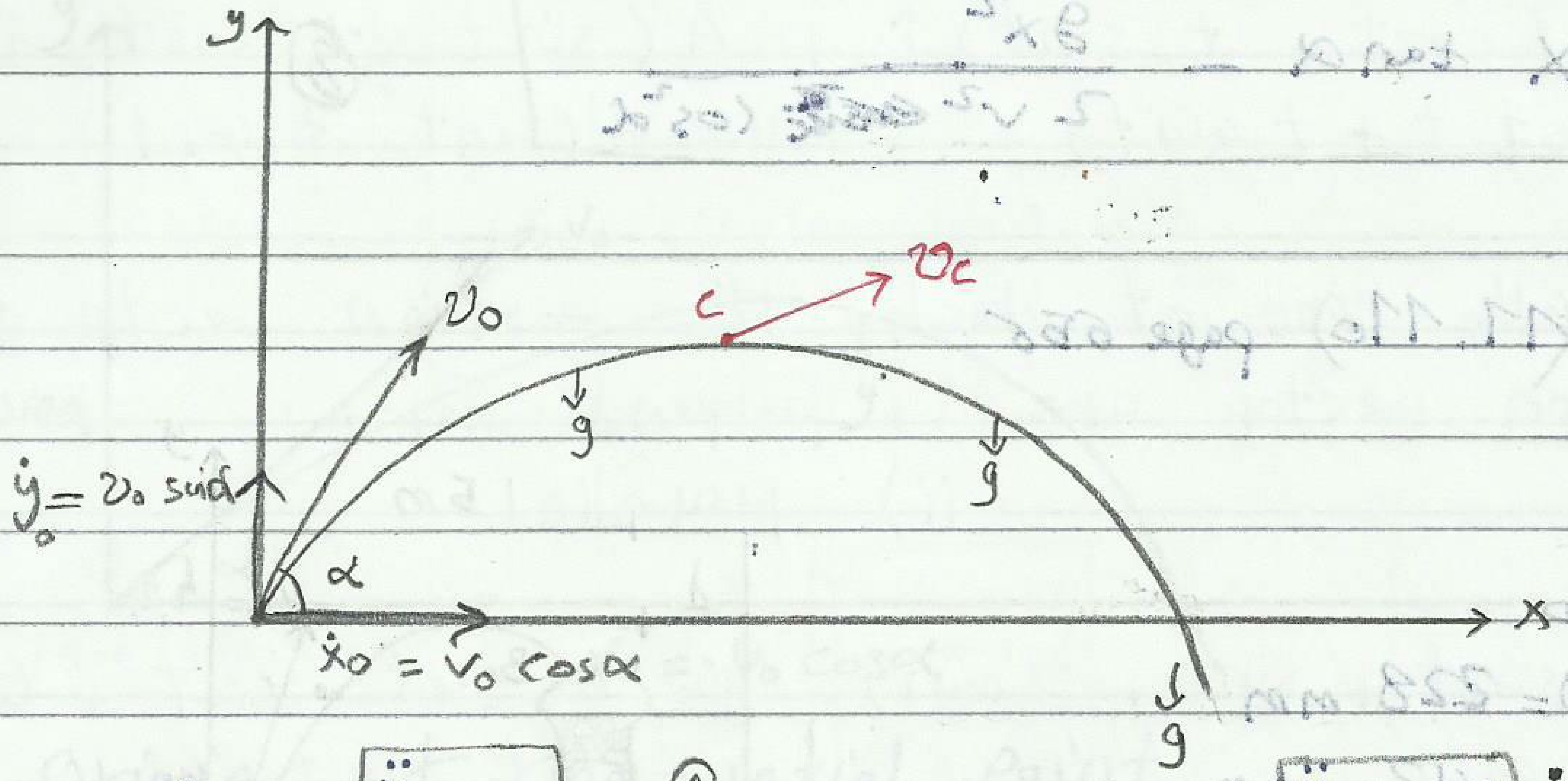
$$1.048 = 4.772 \tan 30 - \frac{9.8 (4.77)^2}{2 V_0^2 \cos 30}$$

$V_0 = V$

$$t_{A-B} = \frac{x}{V_0 \cos\alpha} = \frac{4.772}{V_0 \cos 30}$$

tut. 4/03/2013

projectiles



$a_x = \boxed{\ddot{x} = 0}$ ① $a_y = \boxed{\ddot{y} = -g}$ ②

$V_x = \boxed{\dot{x} = v_0 \cos\alpha}$ ③ $V_y = \boxed{\dot{y} = v_0 \sin\alpha - g\,t}$ ④

$\boxed{x = (v_0 \cos\alpha)\, t}$ ⑤ $\boxed{y = v_0 \sin\alpha \cdot t + \frac{1}{2} g \cdot t^2}$ ⑥

$v^2 = v_0^2 + 2ad \longrightarrow \boxed{\dot{y}^2 = (v_0 \sin\alpha)^2 - 2gy}$ ⑦

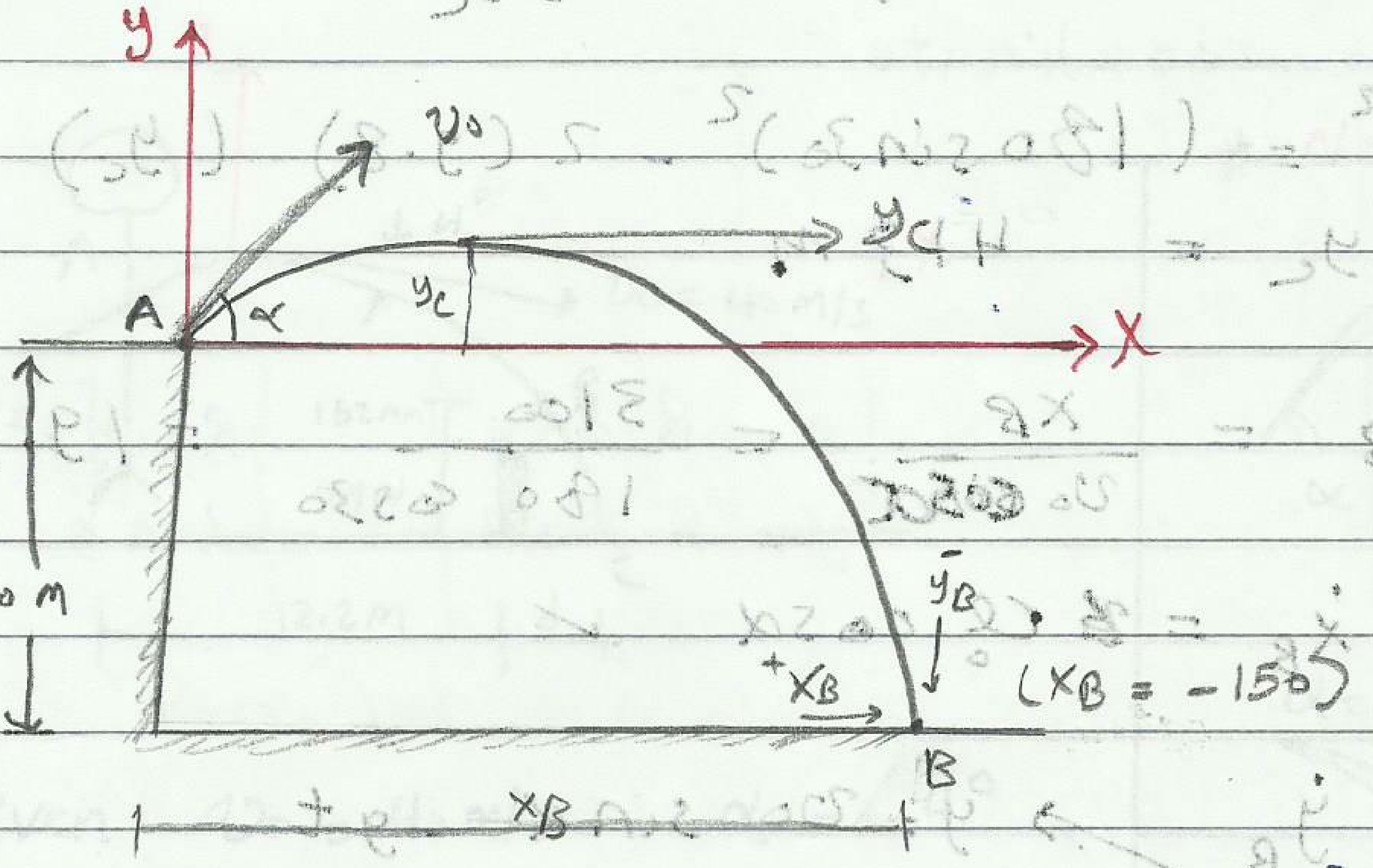
مهم جداً →

path equation $\boxed{y = x \tan\alpha - \frac{g\, x^2}{2 v_0^2 \cos^2\alpha}}$ ⑧

⇐ لو عايز او معطيني T او X تشتغل من ⑥ ← ⑦

⇐ لو اداني ٣ هجيب الرابع من ⑧ $\boxed{x, y, v_0, \alpha}$

* Example (11.7) page 648 مسأله امتحان



Given. $v_0 = 180$ m/s , $\alpha = 30°$

اقصى ارتفاع فوق الارض

find: i) $x_B = ??$, ii) max height above the ground.

iii) $t_{A\to B} = ??$, $\vec{v}_B = ??$

* اول خطوة دايماً لحل مسأله ال projectile ← x,y , origion

$y_B = -150$ m

$$y = x\tan\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha}$$

$$-150 = x_B \tan 30 - \frac{9.8\, x_B^2}{2(180)^2\cos^2 30}$$

$$\overset{A}{\left[\frac{9.8}{2(180)^2\cos^2 30}\right]} x_B^2 - \overset{B}{[\tan 30°]}\, x_B - 150 = 0$$

لو معندكش آلة

$$x_B = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad \begin{cases} x_B = 3100 \text{ m} \\ x_B = -ve \ \times \end{cases}$$

ii) At max height (c) $\dot{y}_c = 0$

$$\begin{cases} v_0 \sin\alpha - g\, t_{A\to c} = 0 \Rightarrow t_{A\to c} = \dfrac{v_0 \sin\alpha}{g} = 9.18 \text{ s} \\ y_c = (v_0 \sin\alpha)\, t_{A\to c} - \dfrac{1}{2}(9.8)\, t_{A\to c}^2 \simeq 413 \text{ m} \end{cases}$$

above ground add (150 m)

<u>or</u>

$$\dot{y}_c^2 = (v_0 \sin\alpha)^2 - 2 g y_c$$

$$0^2 = (180 \sin 30)^2 - 2(9.8)(y_c)$$

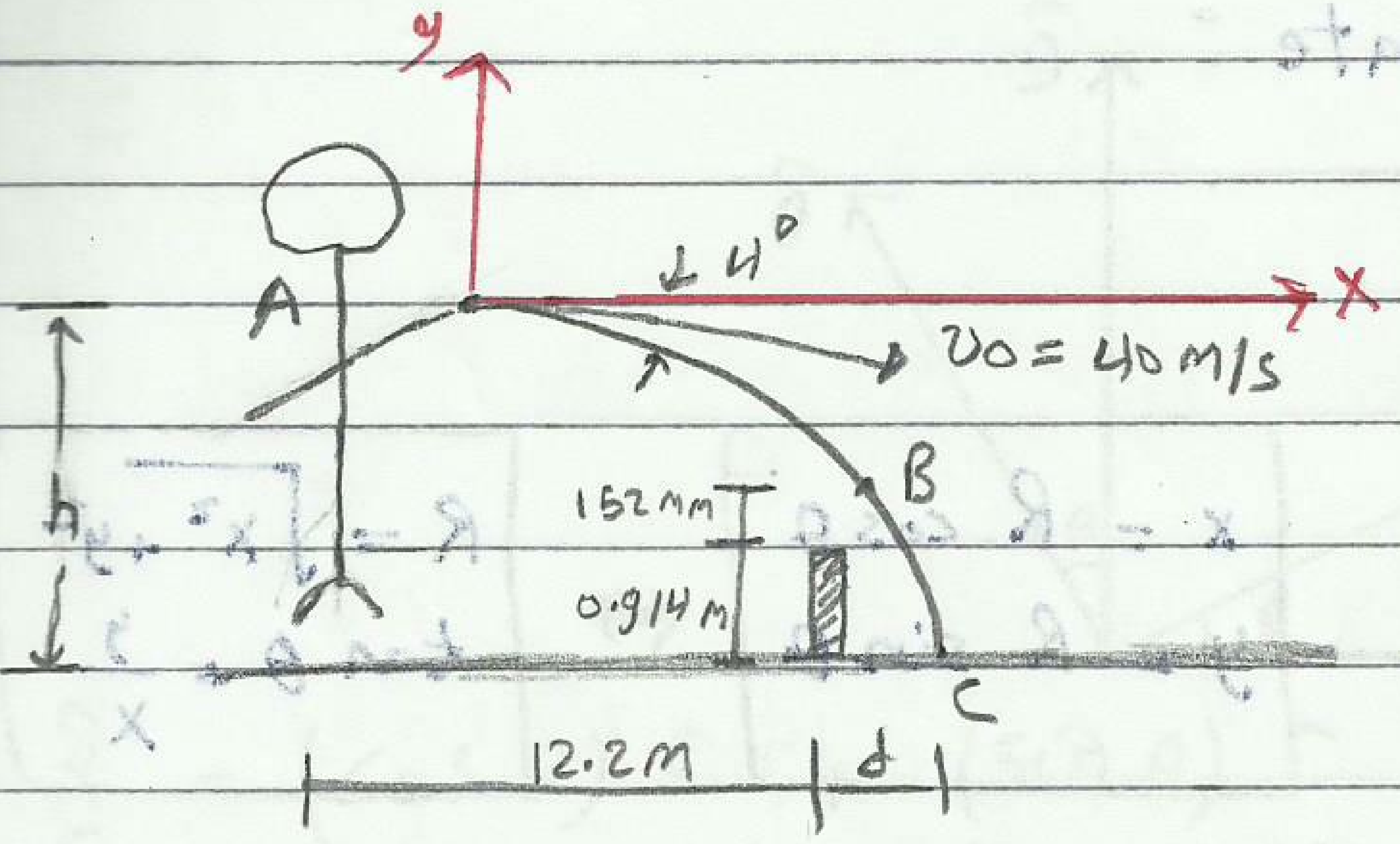
$$\Rightarrow \quad y_c = 413 \text{ m}$$

iii) $t_{AB} = \dfrac{x_B}{v_0 \cos\alpha} = \dfrac{3100}{180 \cos 30} = 19.9 \text{ s}$

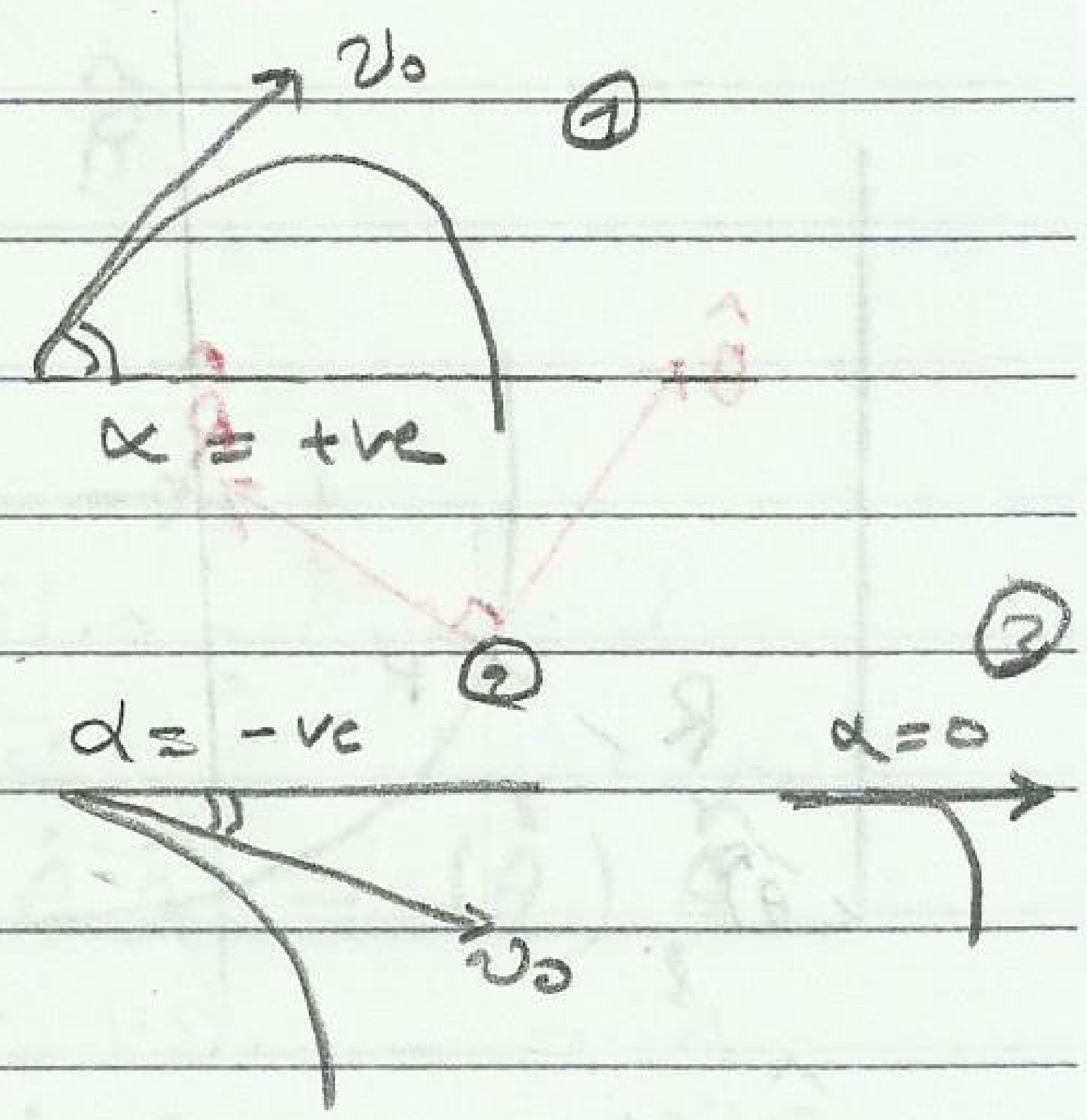
$$\vec{v}_B \begin{cases} \dot{x}_B = \cancel{B}\, v_0 \cos\alpha \checkmark \\ \dot{y}_B \begin{cases} \dot{y} = v_0 \sin\alpha - g t \\ \quad = 180 \sin 30 - 9.8(19.9) \\ \text{or} \\ \dot{y}^2 = (v_0 \sin\alpha)^2 - 2 g y \\ \quad = (180 \sin 30)^2 - 2(9.8)(-150) \end{cases} \end{cases}$$

خلي بالك من الإشارات

Problem (11.106) Page 698



* Note



Given $v_0 = 40$ m/s $\quad \alpha = -4°$

Find $h = ??$, $d = ??$

$$y = x\tan\alpha - \frac{g x^2}{2 v_0^2 \cos^2\alpha}$$

at B

$$y_B = 12.2 \tan(-4°) - \frac{9.8(12.2)^2}{2(40)^2 \cos^2(-4)}$$

$$\boxed{y_B = -1.3\ m}$$

$$h = 0.924 + 0.152 + |y_B| = 2.4\ m$$

at C $\quad y_c = -2.4 = x_c \tan(-4°) - \frac{9.8\, x_c^2}{2(40)^2 \cos(-4°)}$

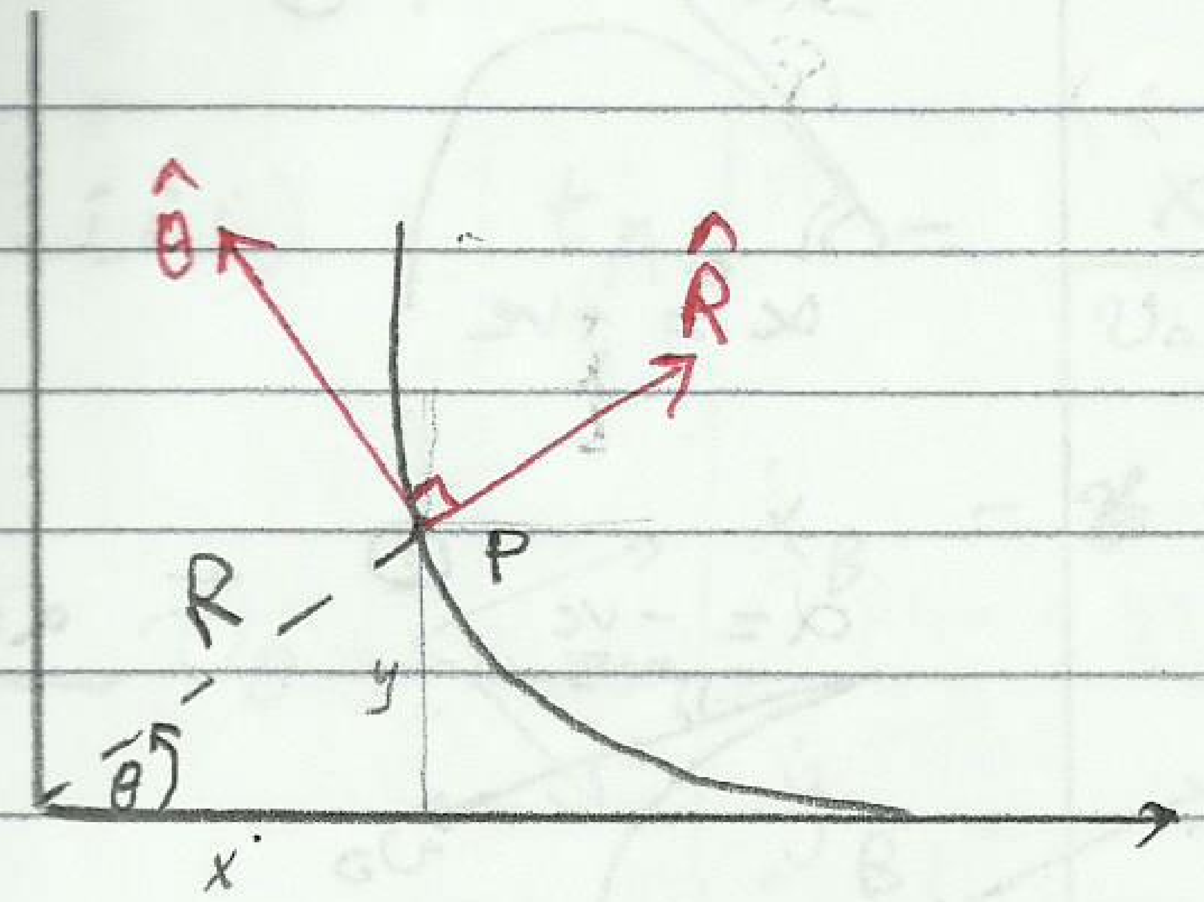
$$d = x_c - 12.2 = \checkmark$$

Assignment: find v_B , v_C

lect. 3

polar & cylindrical coordinate.

planer polar coordinate:



$$\boxed{\begin{aligned} x &= R\cos\theta \\ y &= R\sin\theta \end{aligned}} \Leftrightarrow \boxed{\begin{aligned} R &= \sqrt{x^2+y^2} \\ \tan\theta &= \frac{y}{x} \end{aligned}}$$

coordinates	unit vectors
Radial distance $= R$	Radial unit vector $= \hat{R}$
Transverse angle $= \theta$	Transverse unit vector $= \hat{\theta}$

position: $\boxed{\vec{r} = R\hat{R}}$

velocity: $\vec{v} = \frac{d\vec{r}}{dt} = \dot{R}\hat{R} + R\dot{\hat{R}}$

$$\boxed{\vec{v} = (\dot{R})\hat{R} + (R\dot{\theta})\hat{\theta}}$$

$v_R =$ Radial velocity $\qquad v_\theta =$ transverse velocity

Speed $= \sqrt{v_R^2 + v_\theta^2}$

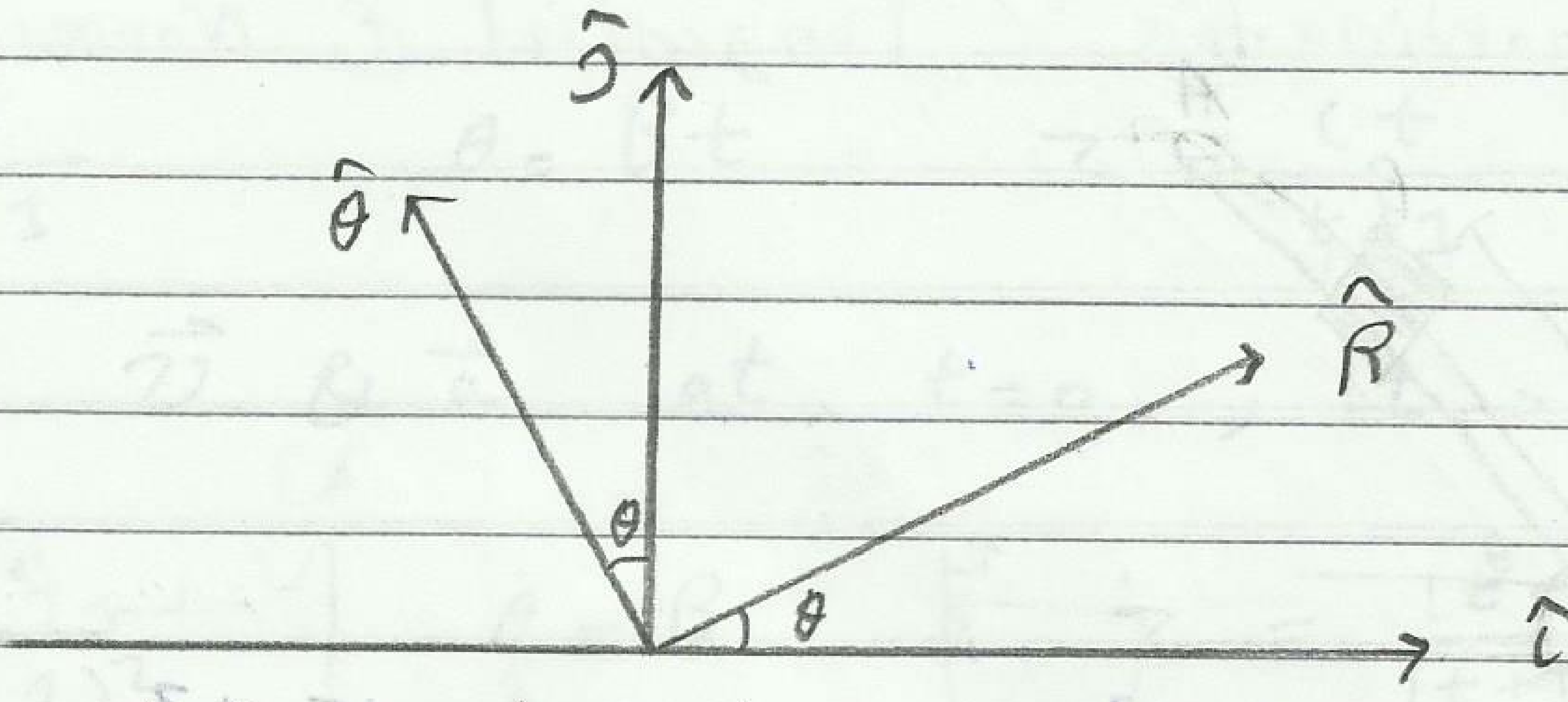
Acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = \ddot{R}\hat{R} + \dot{R}\dot{\hat{R}} + (\dot{R}\dot{\theta})\hat{\theta} + (R\ddot{\theta})\hat{\theta} + (R\dot{\theta})\dot{\hat{\theta}}$

$$\boxed{\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta}}$$

$a_R =$ Radial acceleration $\qquad a_\theta =$ transverse acceleration

$a = \sqrt{a_R^2 + a_\theta^2}$

Time derivative of unit vector.



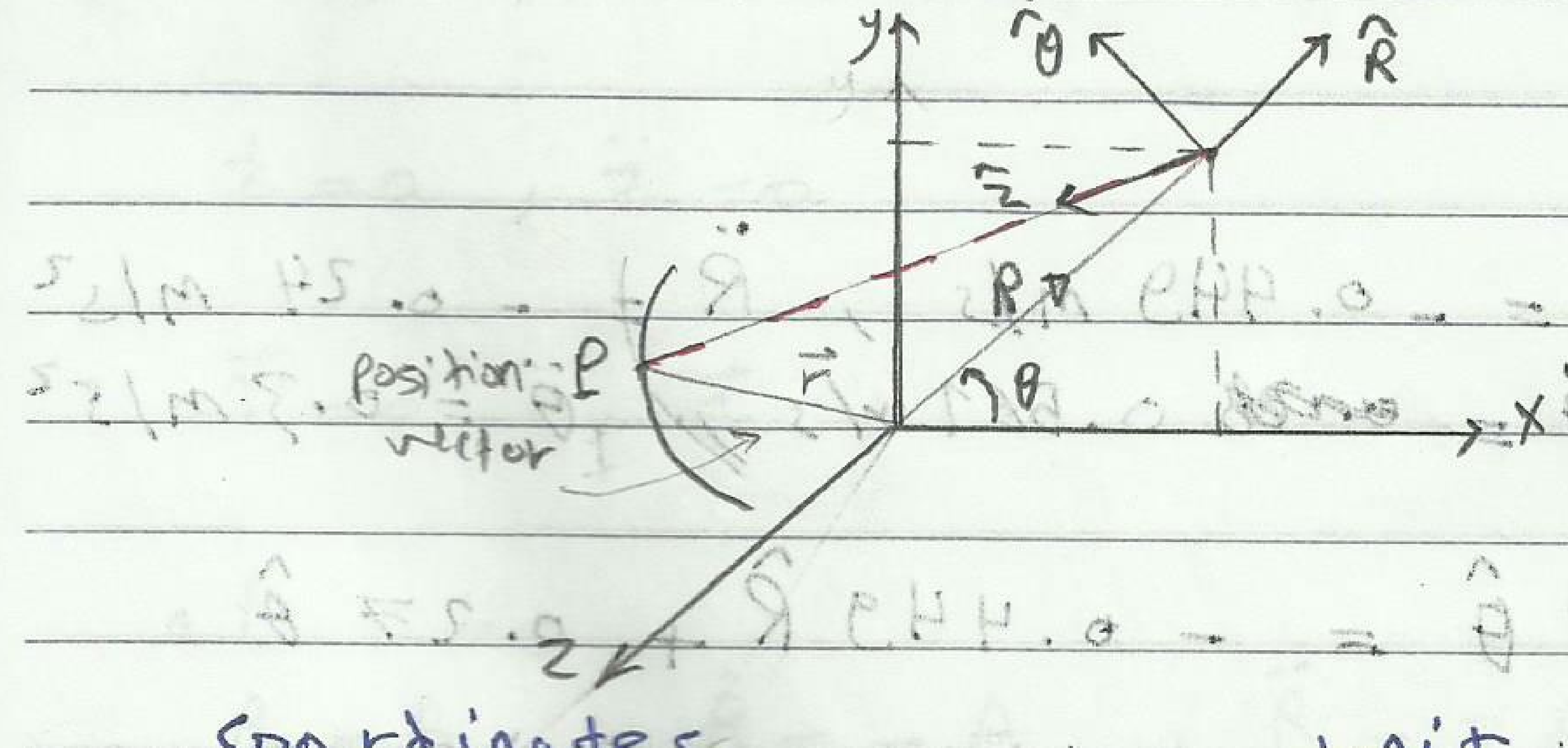
$$\hat{R} = (\cos\theta)\,\hat{i} + (\sin\theta)\,\hat{i}$$

$$\hat{\theta} = -(\sin\theta)\,\hat{i} + (\cos\theta)\,\hat{j}$$

$$\dot{\hat{R}} = \frac{\partial \hat{R}}{\partial t} = (-\sin\theta)\,\dot{\theta}\,\hat{i} + (\cos\theta)\,\dot{\theta}\,\hat{j} = (\dot{\theta})\,\hat{\theta}$$

$$\dot{\hat{\theta}} = \frac{\partial \hat{\theta}}{\partial t} = (-\cos\theta)\,\dot{\theta}\,\hat{i} - (\sin\theta)\,\dot{\theta}\,\hat{j} = -(\dot{\theta})\,\hat{R}$$

space cylindrical coordinate:



coordinates	unit vectors
Radial distance = R	Radial unit vector = $\hat{R}$
Transverse angle = θ	Transverse unit vector = $\hat{\theta}$
Axial distance = z	Axial unit vector = $\hat{K}$

position: $\boxed{\vec{r} = R\hat{R} + z\hat{K}}$

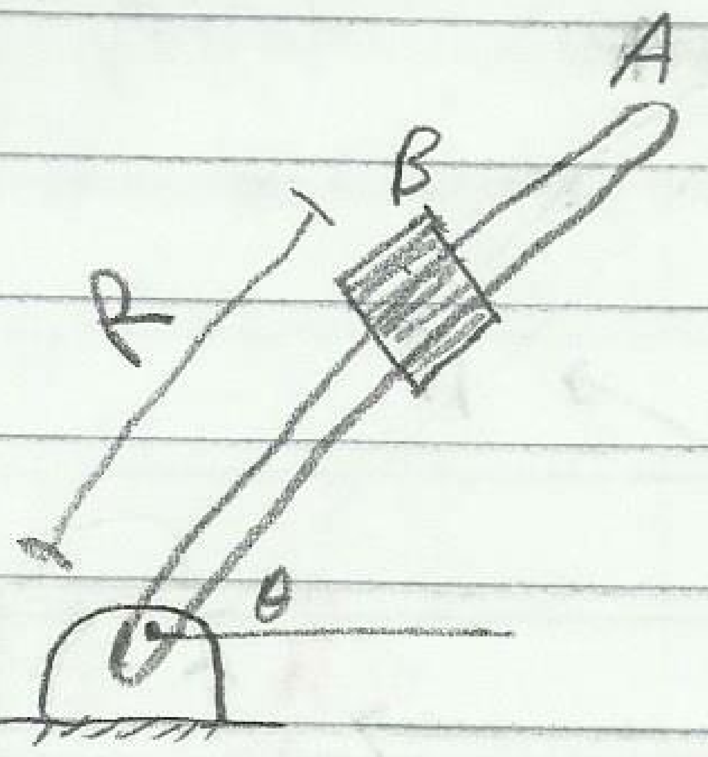
velocity: $\boxed{\vec{v} = (\dot{R})\hat{R} + (R\dot{\theta})\hat{\theta} + \dot{z}\,K}$

speed $= \sqrt{V_R^2 + V_\theta^2 + V_z^2}$

Acceleration: $\boxed{\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{R} + (2\dot{R}\dot{v} + R\ddot{\theta})\hat{\theta} + \ddot{z}\,K}$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_z^2}$$

Example (11-12) page 669



$R = 0.9 - 0.12\, t^2 \qquad \theta = 0.15\, t^2$

calculate: $\vec{\theta}$, $\vec{a}$ at $\theta = 30$

$\dot{R} = -0.24\, t \qquad \dot{\theta} = 0.3\, t$

$\ddot{R} = -0.24 \qquad \ddot{\theta} = 0.3$

let $\theta = 30$

$(30)\left(\frac{\pi}{180}\right)^{3.14} = 0.15\, t^2 \Rightarrow t = 1.87$

at $t = 1.87$

$R = 0.48\ m$, $\dot{R} = -0.449\ m/s$, $\ddot{R} = -0.24\ m/s^2$

$\theta = 0.524$ rad, $\dot{\theta} = 0.561\ r/s$, $\ddot{\theta} = 0.3\ m/s^2$

$\vec{v} = (\dot{R})\hat{R} + (R\dot{\theta})\hat{\theta} = -0.449\,\hat{R} + 0.27\,\hat{\theta}$

$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\,\hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\,\hat{\theta}$

$= -0.391\,\hat{R} - 0.359\,\hat{\theta}$

Problem (11. 180) page 679

$R = \frac{A}{t+1}$ $\theta = Bt$ $z = \frac{ct}{t+1}$

calculate $\vec{v}$ & $\vec{a}$ at $t=0$, $t \to \infty$

$\dot{R} = \frac{-A}{(t+1)^2}$ | $\dot{\theta} = B$ | $\dot{z} = \frac{c(t+1-t)}{(t+1)^2}$

$\ddot{R} = \frac{2A}{(t+1)^3}$ | $\ddot{\theta} = 0$ | $\ddot{z} = \frac{-2c}{(t+1)^3}$

at $t \to \infty$

$R = 0$, $\dot{R} = 0$, $\ddot{R} = 0$

$\dot{\theta} = B$, $\ddot{\theta} = 0$

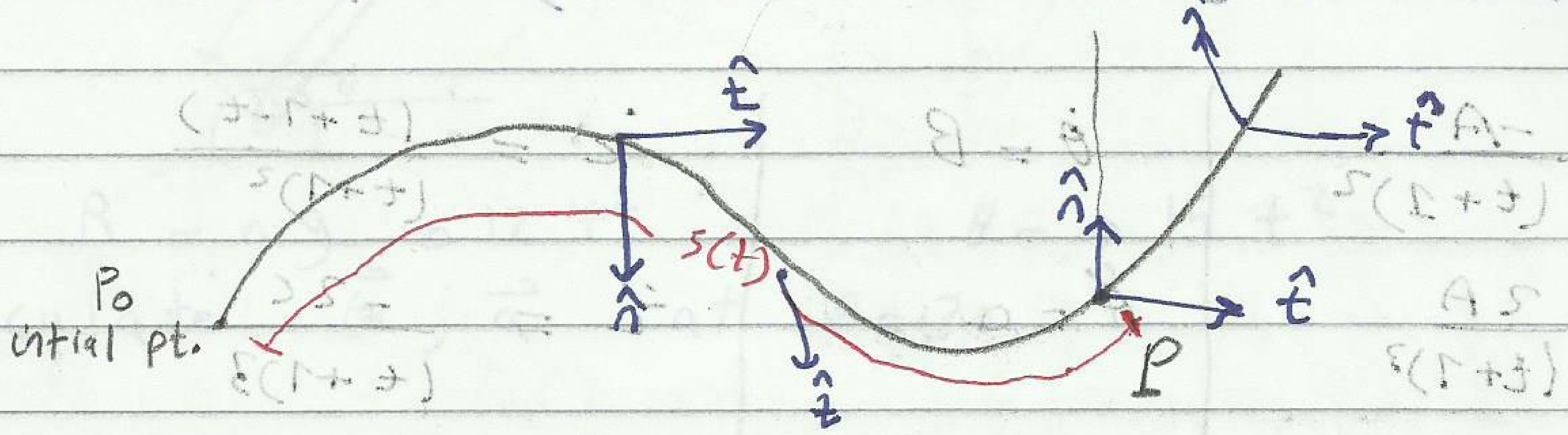
$\dot{z} = 0$, $\ddot{z} = 0$

$\vec{v} = 0$, $\vec{a} = 0$

Curvilinear motion

Intrinsic coordinates (Tangential & Normal coordinates)

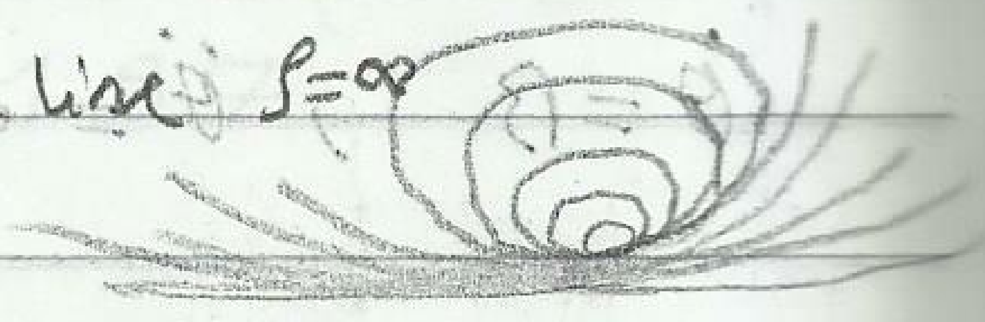
path curve is given



Coordinates:

s : curved travel distance

ρ ; Radius of curvature → circle ρ = constant
→ stright line $\rho = \infty$



unit vectors:

$\hat{t}$; tangential unit vector.

$\hat{n}$; normal unit vector $\perp \hat{t}$ towrds the centre.

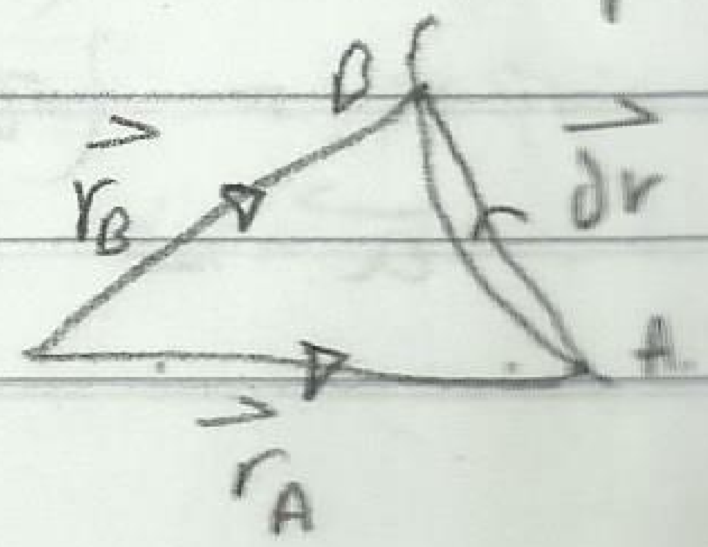
Position: $\vec{r}$ not defined

velocity: $\vec{v} = \frac{dr}{dt}$

$\Delta s \; dt \to 0 \quad \Rightarrow \vec{dr} = (ds)\,\hat{t}$

$\vec{v} = \frac{ds}{dt}\,\hat{t} \Rightarrow \boxed{v = (\dot{s})\,\hat{t}}$ velocity is tangent to path.

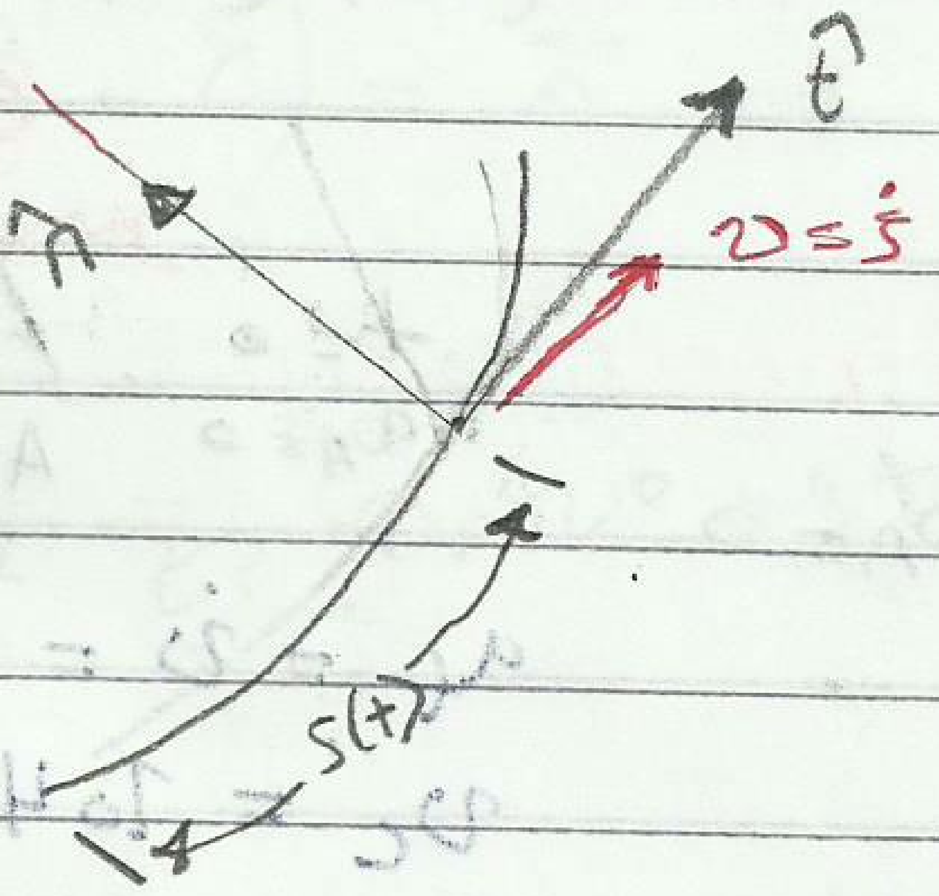
magnitude = speed = $v = \dot{s}$



Acceleration: $\vec{a} = \dfrac{d\vec{v}}{dt} = (\ddot{s})\hat{t} + \dot{s}\,\dot{\hat{t}}$

$$\vec{a} = \dot{v}\,\hat{t} + \dot{s}\left(\frac{v}{\rho}\right)\hat{n}$$

$$\boxed{\vec{v} = v\,\hat{t} = (\dot{s})\,\hat{t}}$$



$$\boxed{\vec{a} = (\dot{v})\,\hat{t} + \dot{s}\left(\frac{v^2}{\rho}\right)\hat{n}}$$

$a_t = \dot{v} = \ddot{s}$ — tangential acc.

$a_n = \dfrac{v^2}{\rho}$ — Normal acc. ← هي مسؤولة عن اتجاه السرعة مع الـ Time

Tangential acc.:
- +ve: v increase
- 0: constant speed
- −ve: v decrease

Normal acc.:
- +ve
- zero: ثابتة, $\rho = \infty$

إذا كانت السرعة ثابتة مقداراً واتجاهاً acceleration $= 0$

$\therefore\ a_t = \dfrac{dv}{dt}$

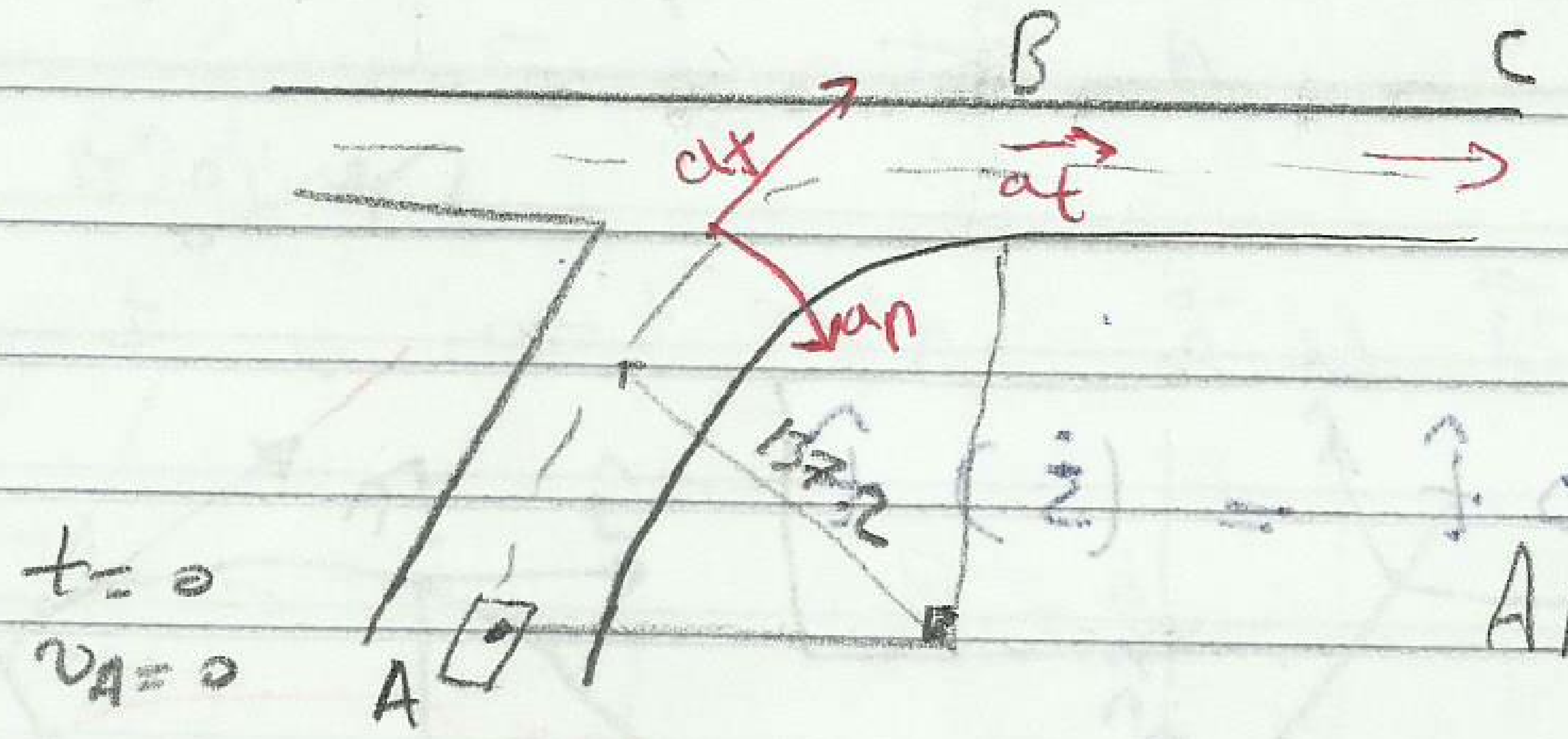
if a_t = constant

$v = v_0 + a t$

$s = v_0 t + \frac{1}{2} a_t t^2$

$v^2 = v_0^2 + 2 a_t s$

Problem (11.142) page 673



$v_c = 104$ Km/hr

AB ≃ 215

$v_A = 0$

$a_t = \dot{v} =$ constant from $A \to C$

$v_c = 104$ Km/h $= 104 \times \frac{1000}{3600}$

$= 104 \times \frac{5}{18} = 28.8$ m/s

Calculate $v_B = ??$ total acceleration at $t = 15$ s ?

$A \to B$

$$v_B^2 = v_A^2 + 2 a_t \, S_{A\to B}$$

$$v_B^2 = 0^2 + 2 a_t \left(\frac{\pi (136.2)}{2} \right) \quad (1)$$

$B \to C$

$$v_c^2 = v_B^2 + 2 a_t \, S_{B\to C}$$

$$(28.8)^2 = v_B^2 + 2 a_t (97.4)$$

$A \to C$

$$(28.8)^2 = 0^2 + 2 a_t \left(\frac{\pi (136.2)}{2} + 97.4 \right)$$

$$\boxed{a_t = 1.365 \text{ m/s}^2}$$

$A \to C$ من A ل C ثابت const. $\therefore a_t$

sub. in (1) or (2)

$v_B = 24.2$ m/s

at $\boxed{t = 15\ s}$

$$s = v_0 t + \tfrac{1}{2} a t^2$$
$$= (0)(15) + \tfrac{1}{2}(7.36)(15)^2$$
$$= 153 \quad \Gamma \; s_{A \to B}$$

on curve

$$a_t = 7.36\ m/s^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(0 + 7.36(15))^2}{137.2} \qquad v = v_0 + a t$$

$$\vec{a} = 7.36\,\hat{t} + \underline{\qquad}\,\hat{n}$$

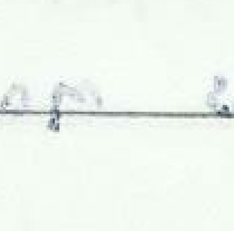
Just before B

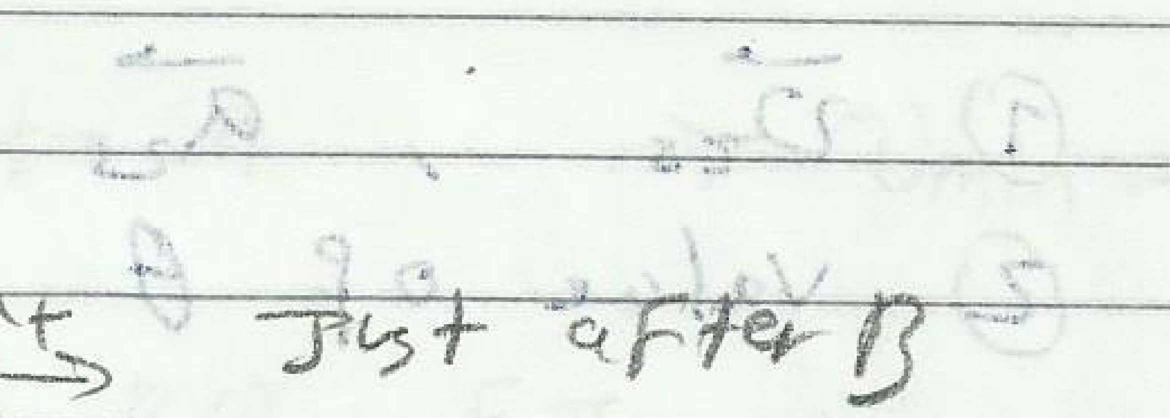
$a_t \qquad \underline{\xrightarrow{a_t}}$ Just after B

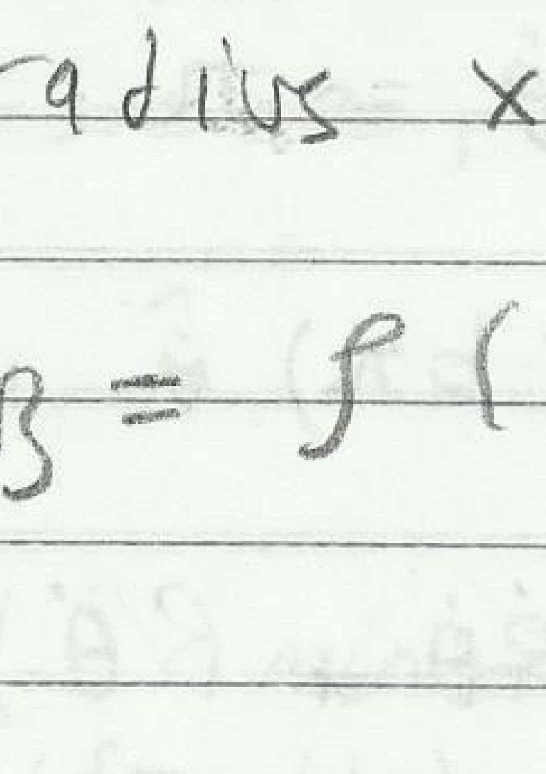
$a_n = \dfrac{V_B^2}{137.2}$

radius × centre angle

$$s_{A \to B} = \rho\left(\frac{\pi}{6}\right)$$







tut.

18/03/2013

polar

$$\vec{r} = (R)\hat{R} + (z)\hat{k}$$

$$\vec{v} = (\dot{R})\hat{R} + (R\dot{\theta})\hat{\theta} + (\dot{z})\hat{k}$$

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta}$$

Intrinsic

$$\vec{v} = (v)\hat{t} = (\dot{s})\hat{t}$$

$$\vec{a} = (\dot{v})\hat{t} + \left(\frac{v^2}{\rho}\right)\hat{n}$$

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problem (11.166) ~~page~~ 677

$R = b(2 + \cos \pi t)$ $\qquad \theta = \pi t$

calculate; ① $\vec{v}_{2s}$, $\vec{a}_{2s}$ at $t = 2s$

② value of θ for which v is max.

$\dot{R} = -b\pi \sin(\pi t)$ $\qquad \dot{\theta} = \pi$

$\ddot{R} = -b\pi^2 \cos(\pi t)$ $\qquad \ddot{\theta} = 0$

at $t = 2s$ $\Rightarrow$ $R = 3b$, $\dot{R} = 0$, $\ddot{R} = -b\pi^2$

$\dot{\theta} = \pi$, $\ddot{\theta} = 0$

$$\vec{v}_{2s} = \cancel{(\dot{R})\hat{R}} + (R\dot{\theta})\hat{\theta} = (3b\pi)\hat{\theta}$$

$$\vec{a}_{2s} = (\ddot{R} - R\dot{\theta}^2)\hat{R} + \cancel{(2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta}}$$

$$= (-b\pi^2 - 3b\pi^2)\hat{R} = -(4b\pi^2)\hat{R}$$

for max. & min speed

$$v = \sqrt{\dot{R}^2 + (R\dot{\theta})^2} = \sqrt{b^2\pi^2 \sin^2(\pi t) + b^2\pi^2(2+\cos\pi t)^2}$$

$$= b\pi\sqrt{\sin^2\pi t + \cos^2\pi t + 4 + 4\cos(\pi t)}$$

$$\boxed{v = b\pi\sqrt{5 + 4\cos\pi t}}$$

method ①

$$\frac{dv}{dt} = 0 = \frac{b\pi \cdot (\ldots)}{2\sqrt{\ }}$$

method ②

v_{min} at $\cos\pi t = -1$ $\Rightarrow v_{min} = b\pi$

$\theta = \pi t = \pi, 3\pi, 5\pi$

$t = 1, 3, 5, \ldots$ s

v_{max} at $\cos\pi t = 1$ $\Rightarrow v_{max} = 3b\pi$

$\theta = \pi t = 0, 2\pi, 4\pi$

$t = 0, 2, 4, 6, \ldots$ s

لو فيه $\cos^2\pi t$ ←

min $= 0$ max $= 1$

لو $\sqrt{5 - 4\cos\pi b}$ ←

min $= 1$ max $= -1$

Note : $\vec{v}\cdot\vec{a} = v_R a_R + v_\theta a_\theta + v_z a_z$

$$\vec{v}\times\vec{a} = \begin{vmatrix} \hat{R} & \hat{\theta} & \hat{k} \\ v_R & v_\theta & v_z \\ a_R & a_\theta & a_z \end{vmatrix}$$

---

problem (11-181) page 679

$R = A \qquad \theta = 2\pi t \qquad z = \frac{A t^2}{4}$

calculate $\vec{v}$, $\vec{a}$
& the time at which $\vec{v} \perp \vec{a}$

$\dot{R} = 0 \qquad \dot{\theta} = 2\pi \qquad \dot{z} = \frac{At}{2}$

$\ddot{R} = 0 \qquad \ddot{\theta} = 0 \qquad \ddot{z} = \frac{A}{2}$

$$\vec{v} = (\dot{R})\hat{R} + (R\dot{\theta})\hat{\theta} + \dot{z}\hat{k}$$
$$= (2\pi A)\hat{\theta} + \left(\frac{At}{2}\right)\hat{k}$$

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta} + \ddot{z}\hat{k}$$
$$= (-4\pi^2 A)\hat{R} + \left(\frac{A}{2}\right)\hat{k}$$

$\vec{v} \perp \vec{a}$ when $\vec{v}\cdot\vec{a} = 0$

$$v_R a_R + v_\theta a_\theta + v_z a_z = 0$$
$$(0)(-4\pi^2 A) + (2\pi A)(0) + \left(\frac{At}{2}\right)\left(\frac{A}{2}\right) = 0$$

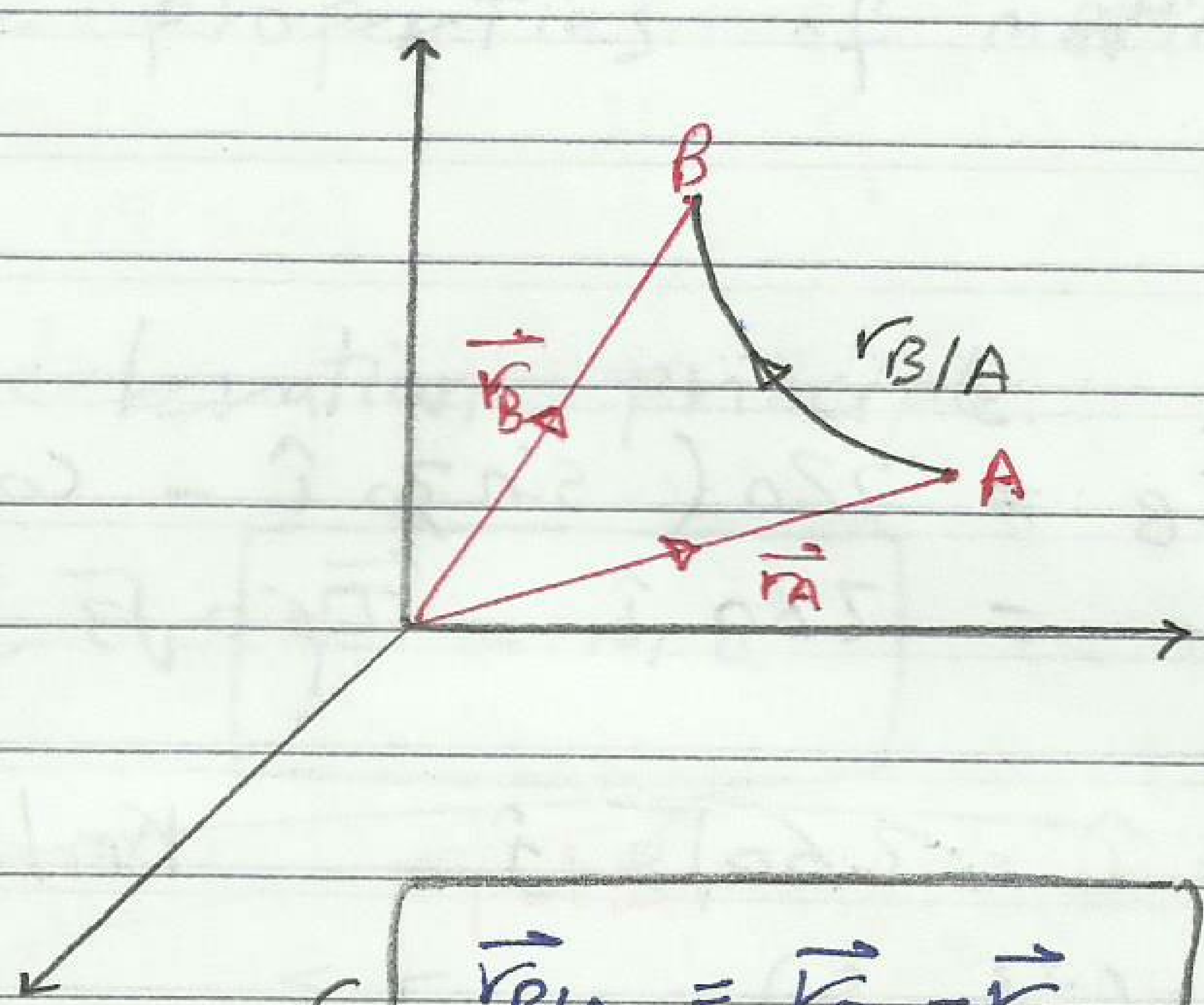
$$\frac{A^2}{4} t = 0 \Longrightarrow t = 0$$

A can't = 0

$\vec{v} \parallel \vec{a}$ اعجز $\Longleftarrow$ $\vec{v}$ can't $\parallel \vec{a}$ لو طلب ان

مسألة حلها غير منطقية

Relative motion



$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

الـ B بالنسبة الى A

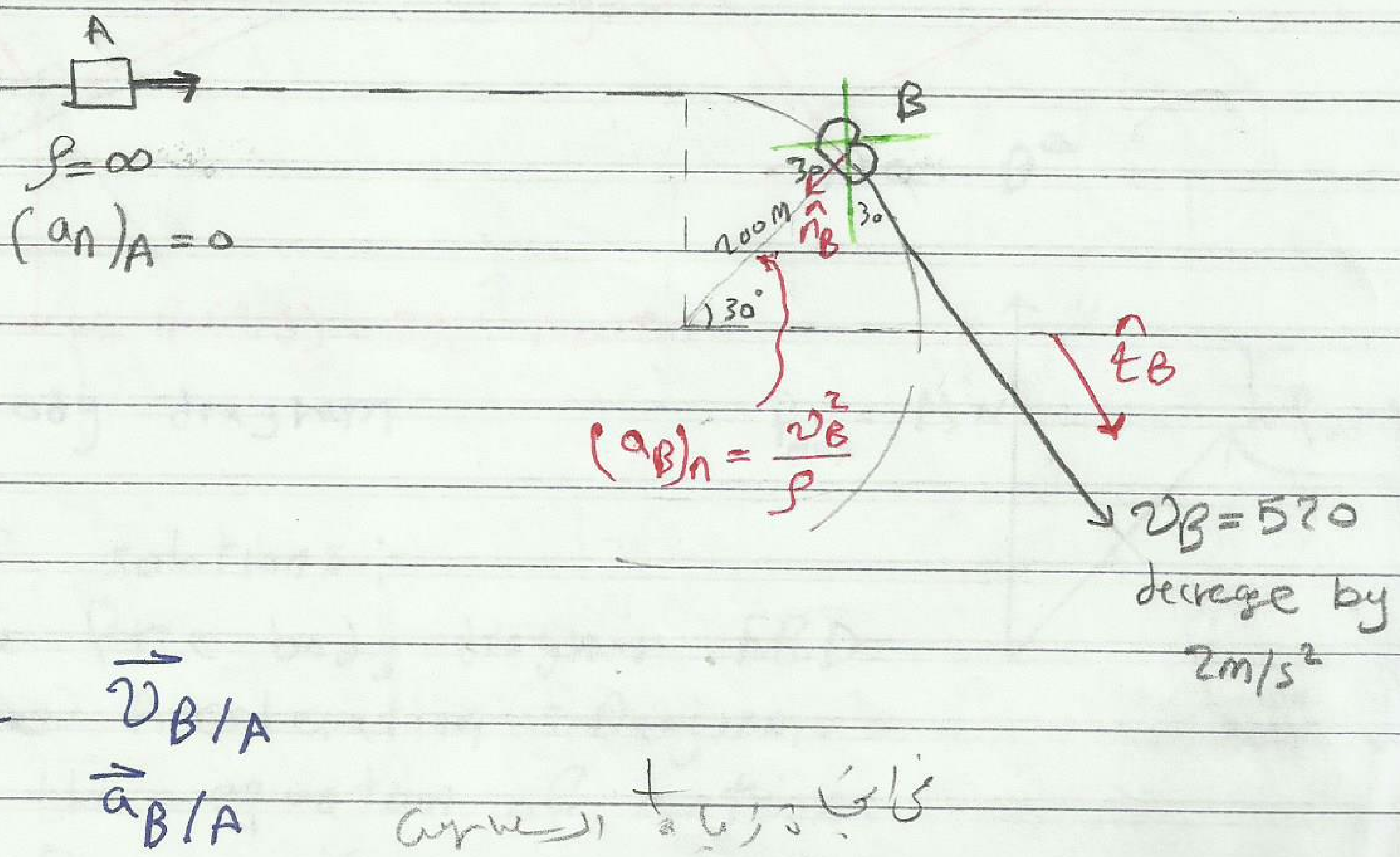
$$\boxed{\begin{aligned} \vec{r}_{B/A} &= \vec{r}_B - \vec{r}_A \\ \vec{v}_{B/A} &= \vec{v}_B - \vec{v}_A \\ \vec{a}_{B/A} &= \vec{a}_B - \vec{a}_A \end{aligned}}$$

$\vec{v} = v\hat{t} = \dot{s}\hat{t}$

$\vec{a} = \dot{v}\hat{t} + \left(\frac{v^2}{\rho}\right)\hat{n}$

problem (11.143) page 673

$v_A = 420$ Km/hr increase by $6 m/s^2 = a_t = a$



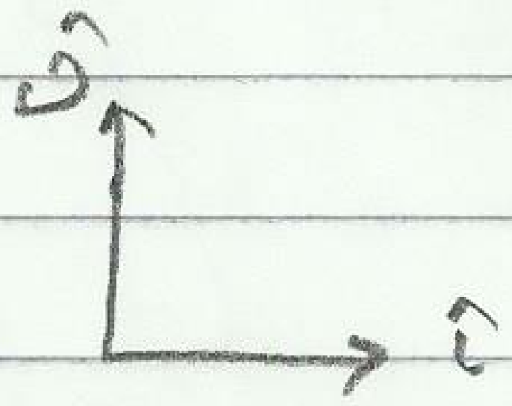
calculate $\vec{v}_{B/A}$

$\vec{a}_{B/A}$ كأني راكب t السابقة

عن اهلها وازاي دوم

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$



$$\vec{v}_A = 420\,\hat{i}$$

$$\hat{v}_B = 520\,\hat{t}_B = 520(\sin 30\,\hat{i} - \cos 30\,\hat{j})$$
$$= 260\,\hat{i} - 260\sqrt{3}\,\hat{j}$$

$$\vec{v}_{B/A} = -160\,\hat{i} - 260\sqrt{3}\,\hat{j} \quad \text{km/hr}$$

$$\vec{a}_A = 6\,\hat{i}$$
$$\vec{a}_B = -2\,\hat{t}_B + \frac{(520 * \frac{5}{18})^2}{200}\,\hat{n}_B$$

$$= -2(\sin 30\,\hat{i} - \cos 30\,\hat{j}) + \frac{(520 * \frac{5}{18})^2}{200}(-\cos 30\,\hat{i} - \sin 30\,\hat{j})$$

chapter 12

Kinetics of aparticle

Relates the properties of motion & its reasons (force)

force - Acceleration principle. قانون نيوتن الثاني

$$\boxed{\vec{F} = m\vec{a}}$$

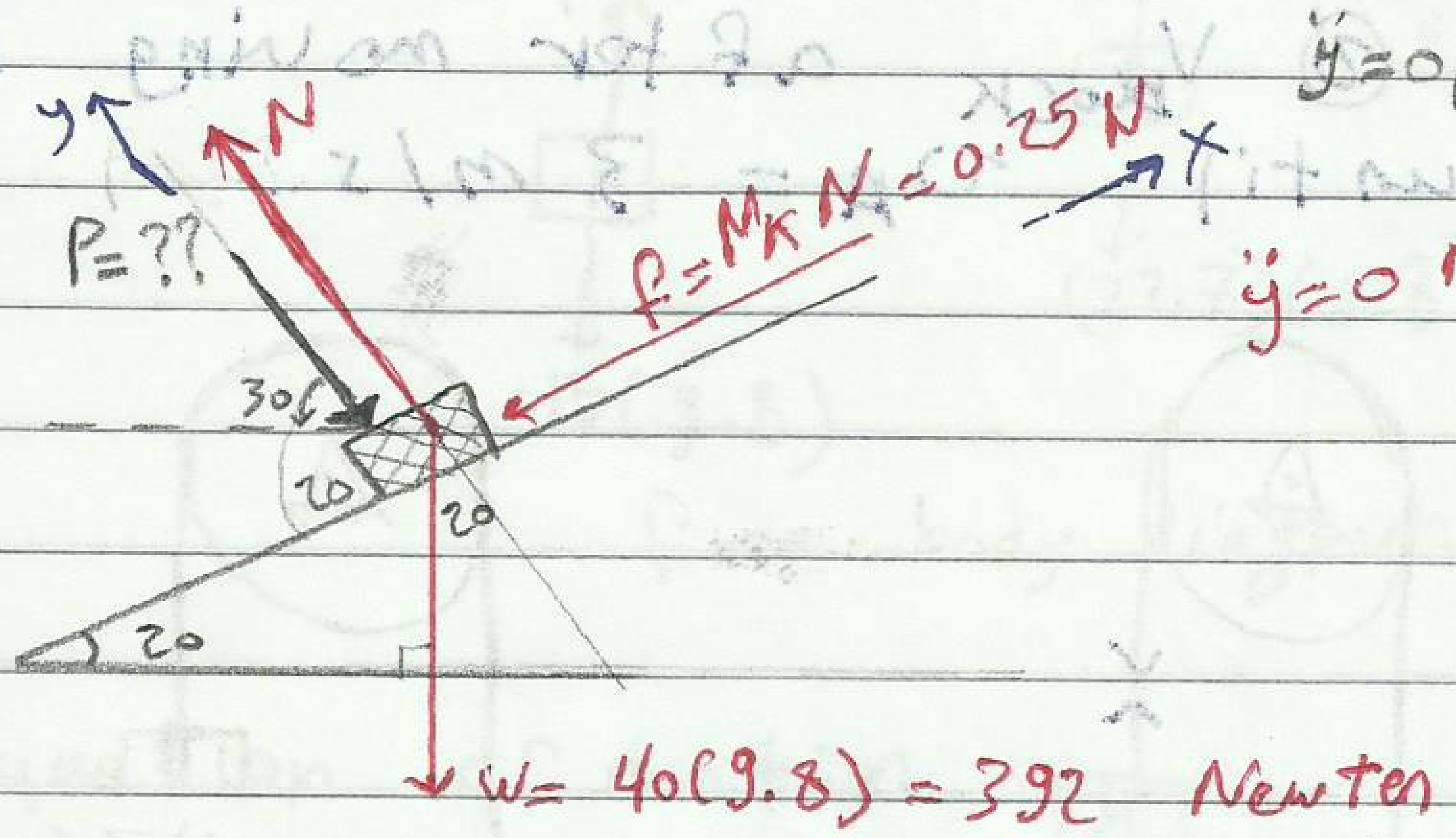
cartesian	polar	Intrisic
$\sum F_x = m\ddot{x}$	$\sum F_R = m(\ddot{R} - \dot{\theta}^2)$	$\sum f_t = m\dot{v}$
$\sum f_y = m\ddot{y}$	$\sum f_\theta = m(2\dot{R}\dot{\theta} + R\ddot{\theta})$	$\sum f_n = m\left(\frac{v^2}{\rho}\right)$

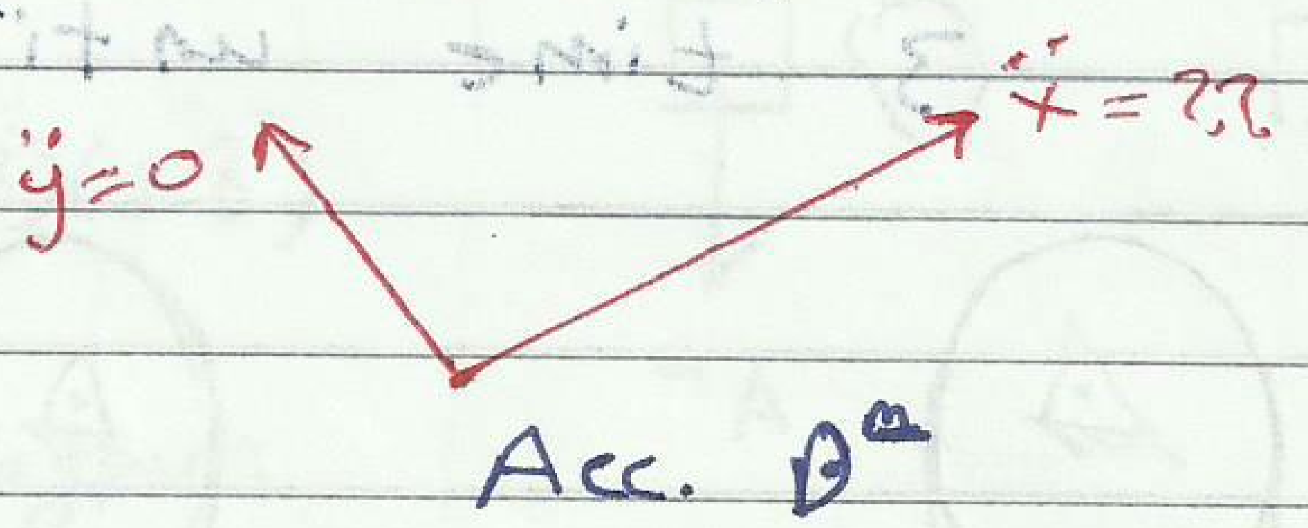
problem (12-10) page 707

$m = 40$ Kg $\quad v_0 = 0 \quad \mu_s = 0.3 \quad \mu_K = 0.25$

calculate $P = ??$ such that the block moves 10 m into 4s.

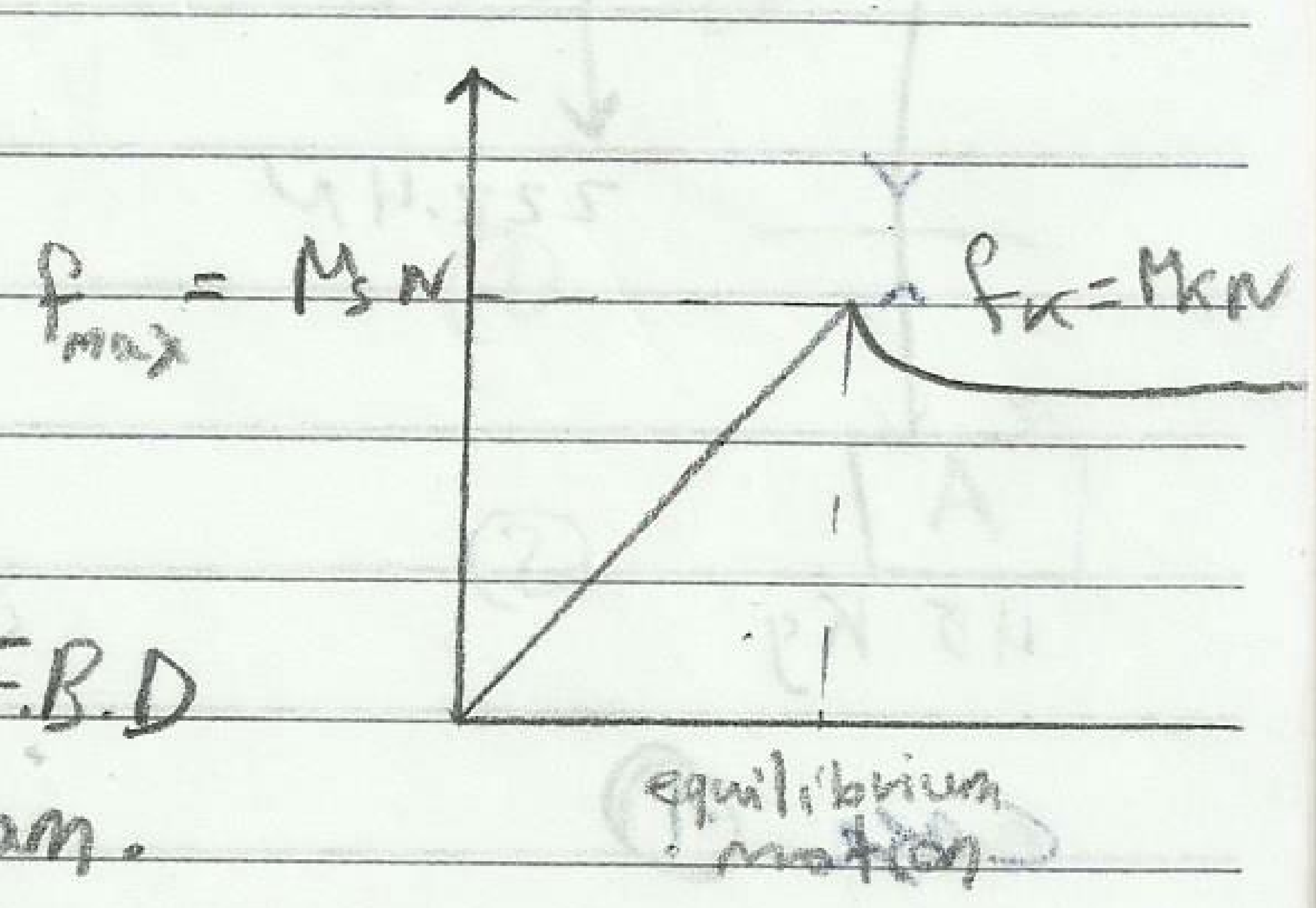
طالما قال إنه بيتحرك على plane يبقى $\ddot{y} = 0$





Acc. D

free body diagram



steps of solutions:-

1. Draw the free body diagram F.B.D
2. Draw the Acceleration s Diagram.
3. write the equation of motion.
4. solve for unknowns.

equation of motion

$\sum F_x = m\ddot{x} \Rightarrow P\cos 50 + 392\sin 20 - 0.25N = 40a$ — ①

$\sum F_y = m\ddot{y} \Rightarrow N - P\sin 50 - 392\cos 20 = 40a$ — ②

a constant ⟵ v constant
a varible ⟵ v varible

since the forces are constant

$\Rightarrow \therefore a = \text{constant}$ (Rectilinear)

$d = v_0 t + \frac{1}{2} a t^2$

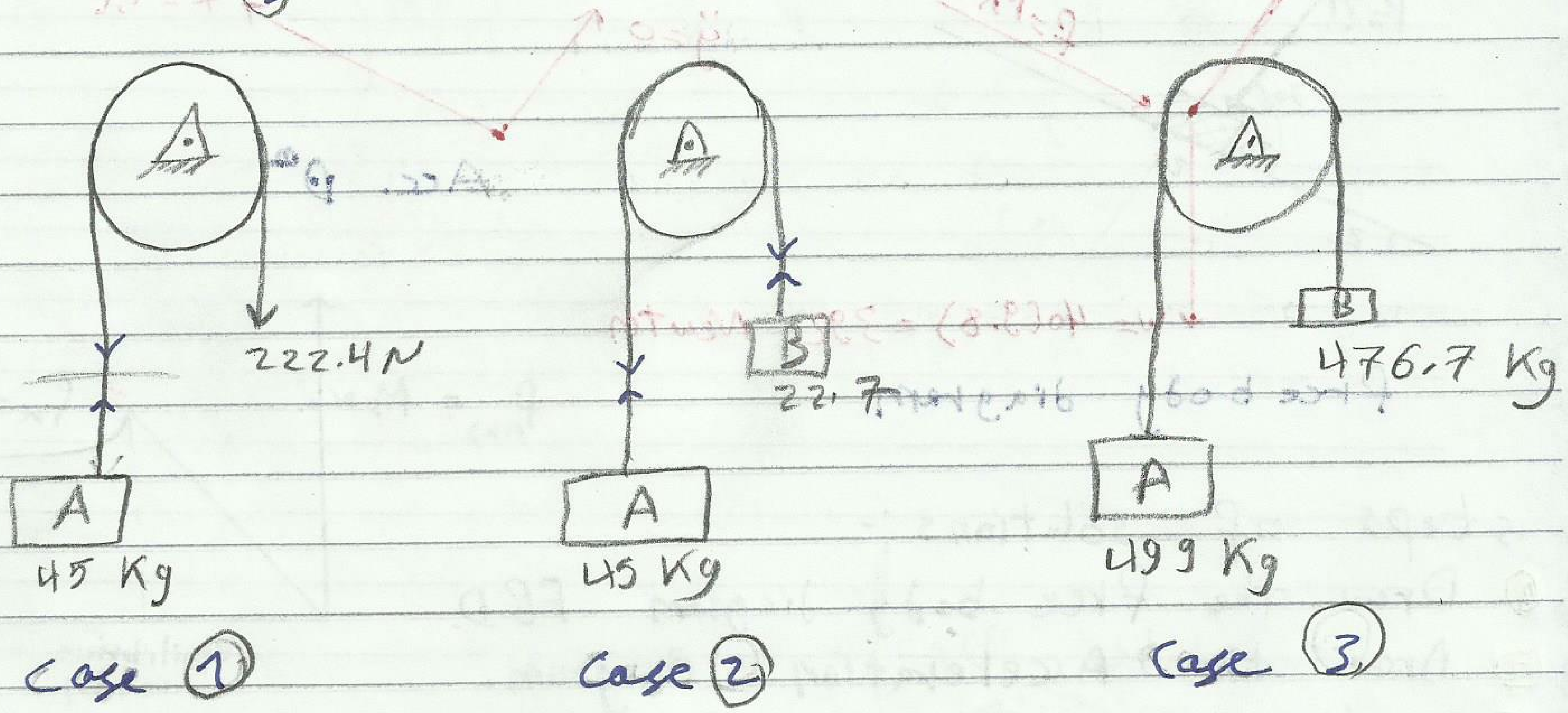
$10 = (0)(4) + \frac{1}{2} a (4)^2 \Rightarrow a = 1.25\ m/s^2$

from ①, ② get P, N

---

problem (12.20) page 708

calculate ① $a_{block\,A}$, ② V_{block} after moving 1.5 m
③ time until $v_A = 3\ m/s$.

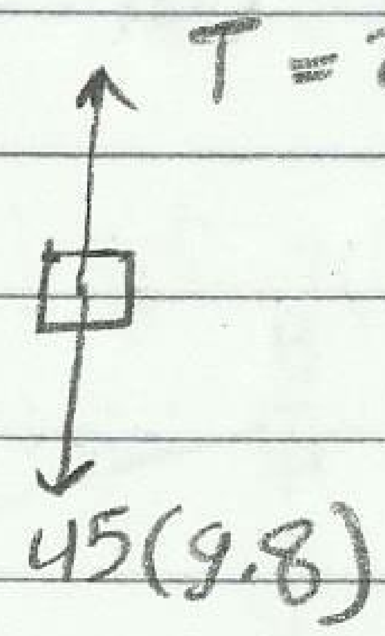


Case ① Case ② Case ③

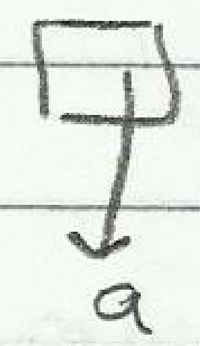
غير

02/04/2013

Case (1)



free body Diagram



Acc. D

Equation of motion:

$45(9.8) - 222.4 = 45\,a \Rightarrow a = \text{---}\ m/s^2$

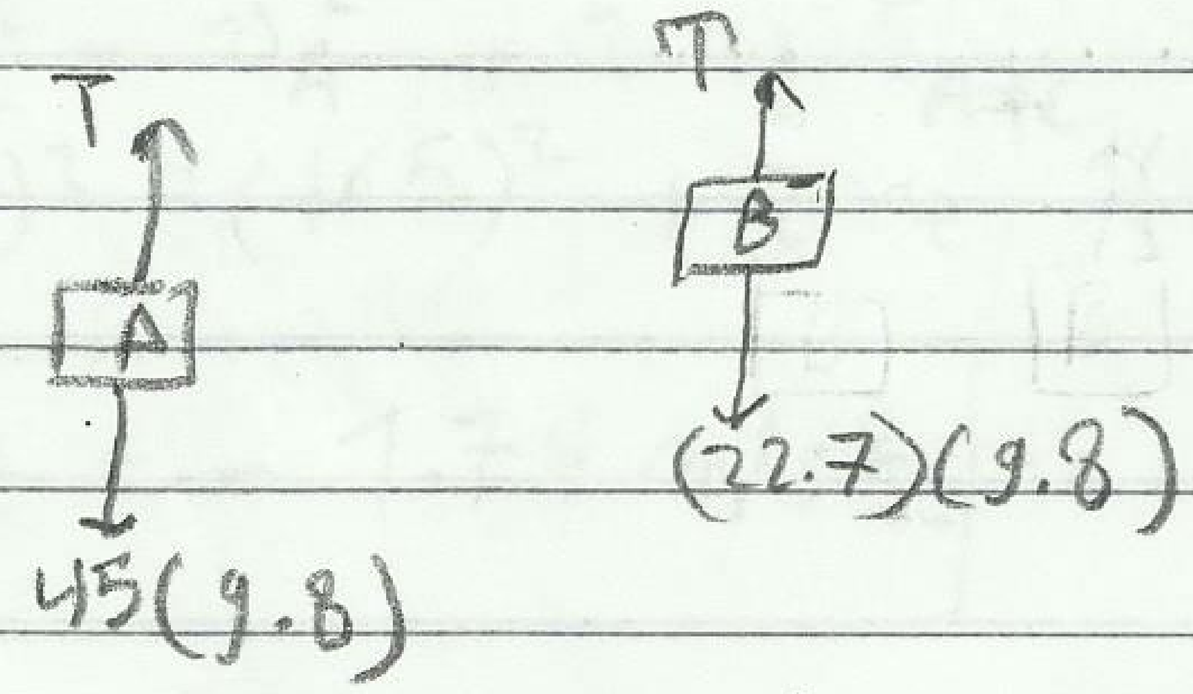
$v^2 = v_0^2 + 2ad = 0^2 + 2(\text{----})(1.5)$

$v_A = \text{----}\ m/s$

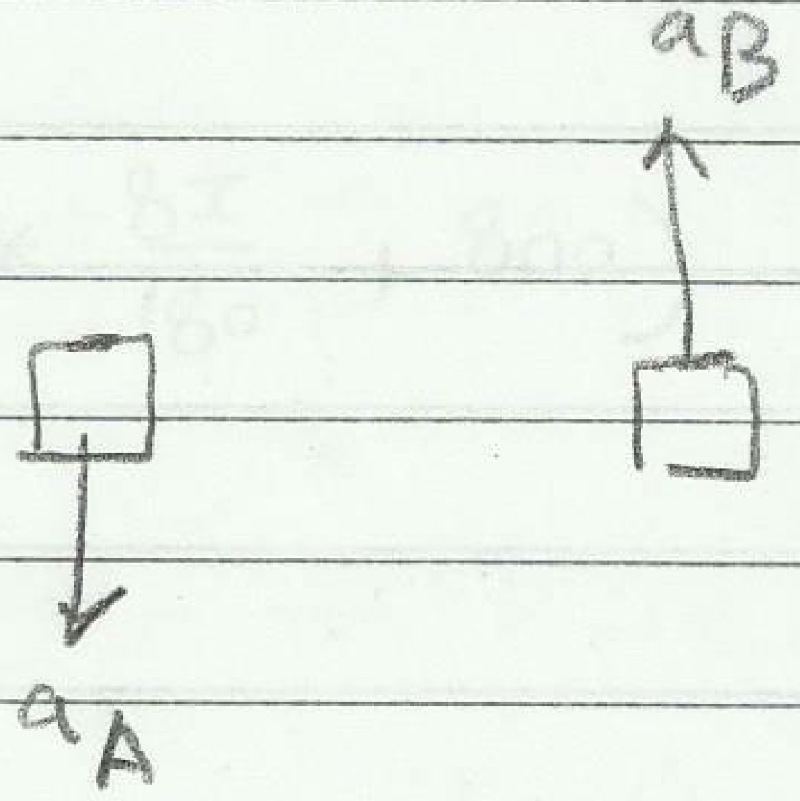
$v = v_0 + at \Rightarrow 3 = 0 + (\text{---})\,t$

$t = \checkmark$

Case (2)



free body diagram



acc. D

Equation of motion

A: $(45)(9.8) - T = 45\,a_A$ —— (1)

B: $T - (22.7)(9.8) = 22.7\,a_B$ —— (2)

$\boxed{a_A = a_B}$

$m_A g - m_B g = (m_A + m_B)\,a_A$

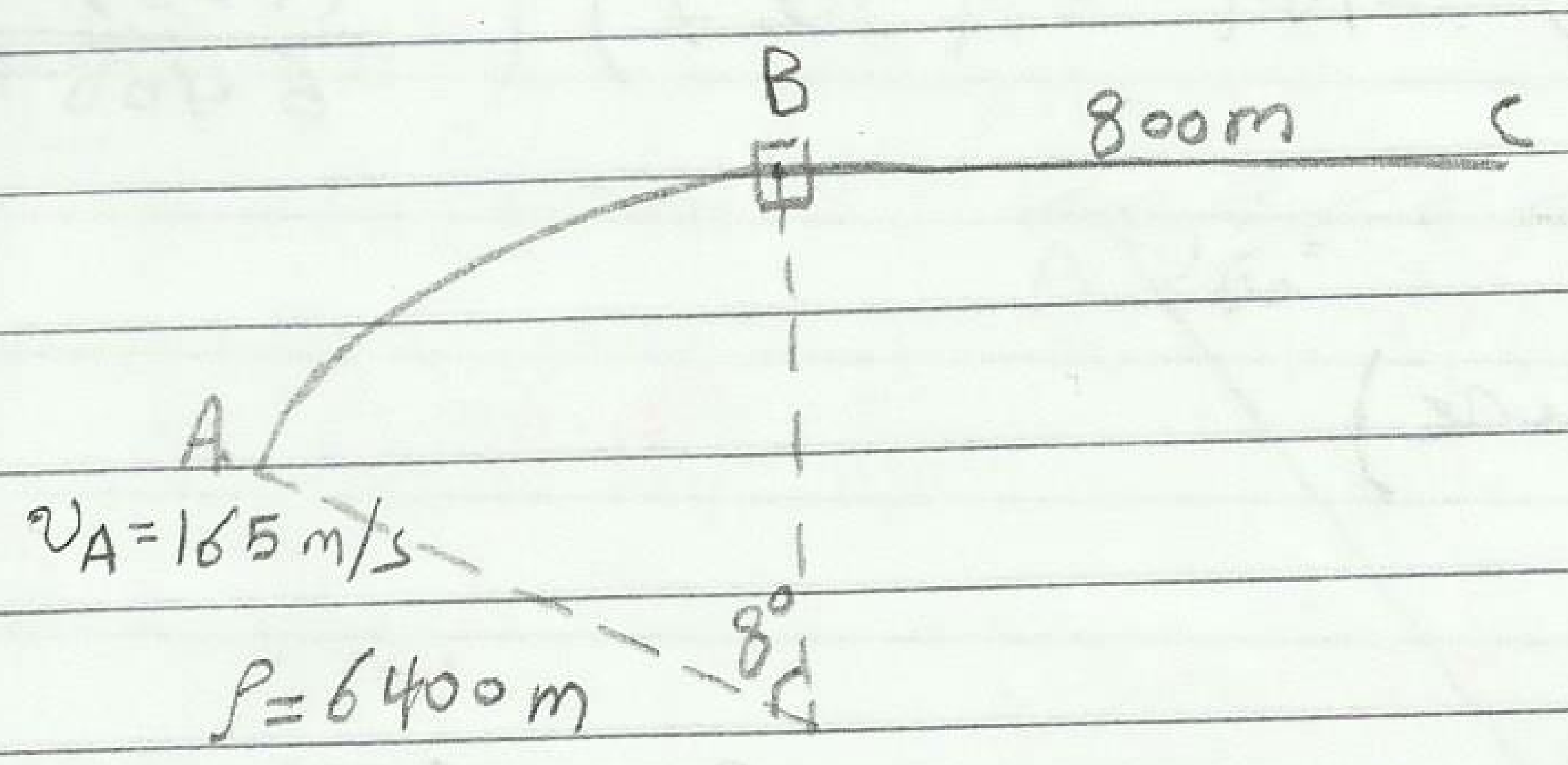
T لازم تطلع لما نجمع (22.7)(9.8) و 45(9.8)

tut.

~~01/04/2013~~

problem (12.46) page 713

اوعى [illegible] ⇓



$v_C = 146\ m/s$

$a_t =$ constant

the spead decrease with a constant rate. calculate the change in forces on a passenger $m = 90.7$ at B.

tut.

$\Sigma F_t = m\dot{v}$, $\Sigma F_n = m\left(\frac{v^2}{\rho}\right)$

solution:

kinematics

$\because a_t =$ constant

$\therefore v_C^2 = v_A^2 + 2 a_t S_{A\to C}$

$(146)^2 = (165)^2 + 2 a_t \left(6400 * \frac{8\pi}{180} + 800\right)$

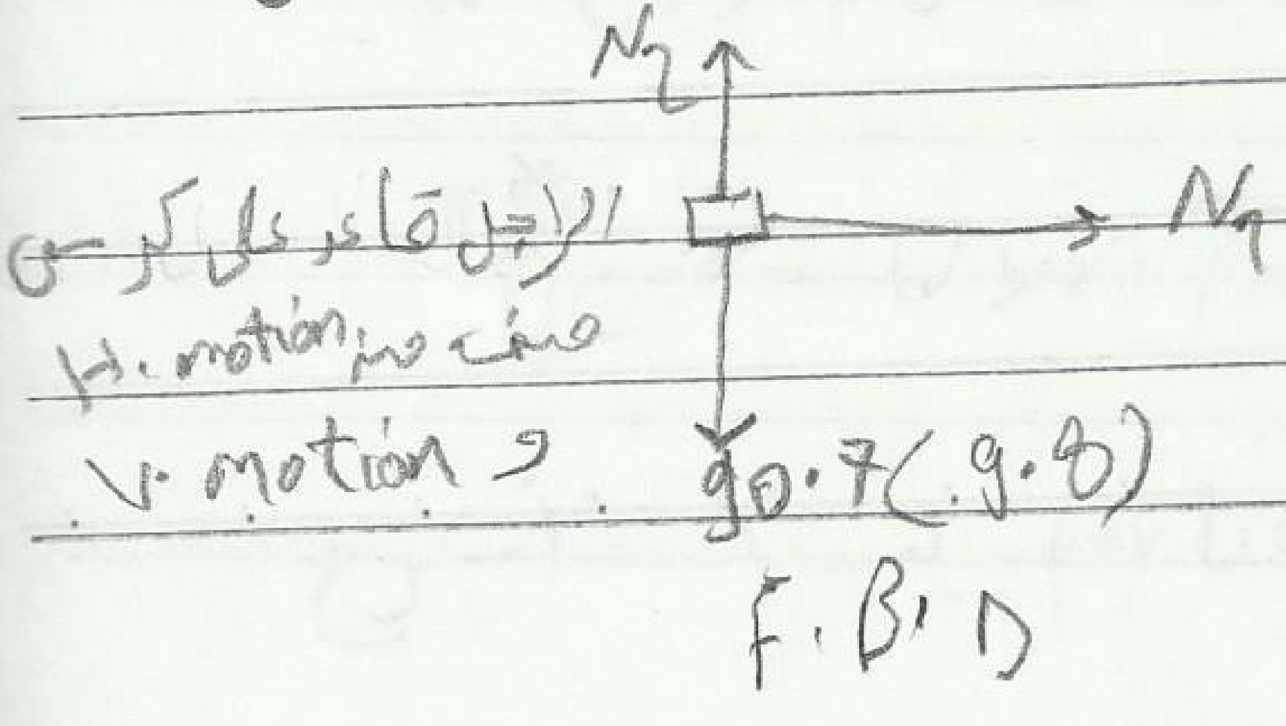
$$\boxed{a_t = -1.74\ m/s^2}$$

$v_C^2 = v_B^2 + 2 a_t S_{B\to C}$

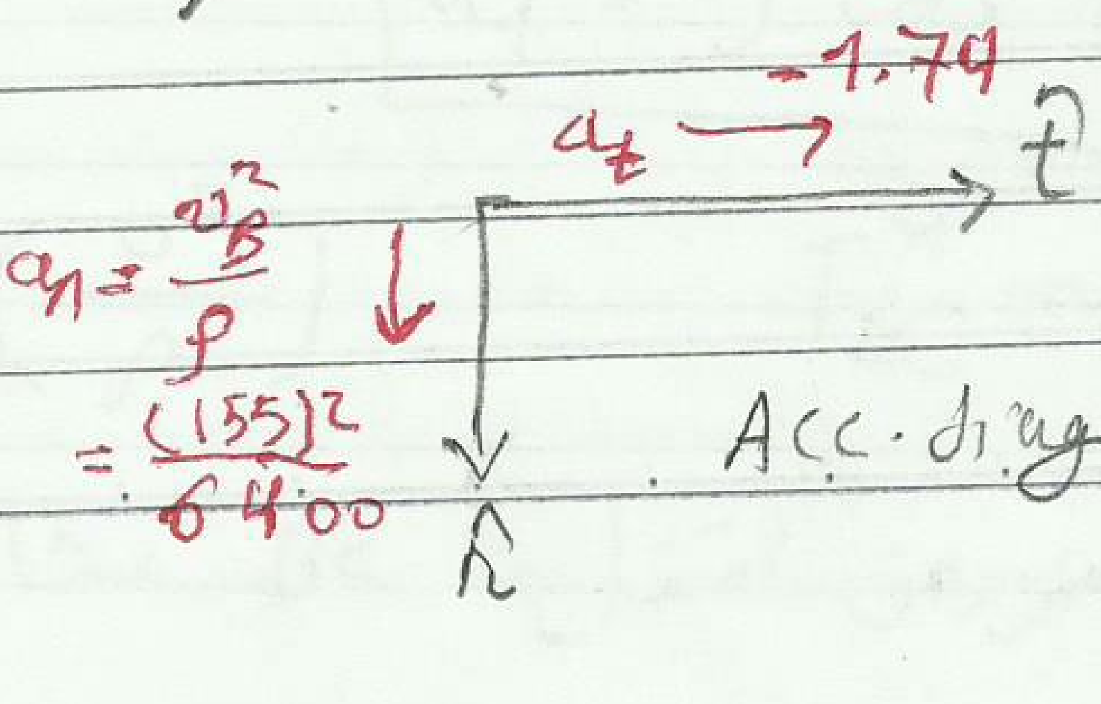
$(146)^2 = v_B^2 + 2(-1.74)(800)$

$$\boxed{v_B = 155\ m/s}$$

Just before B (curve)



F.B.D

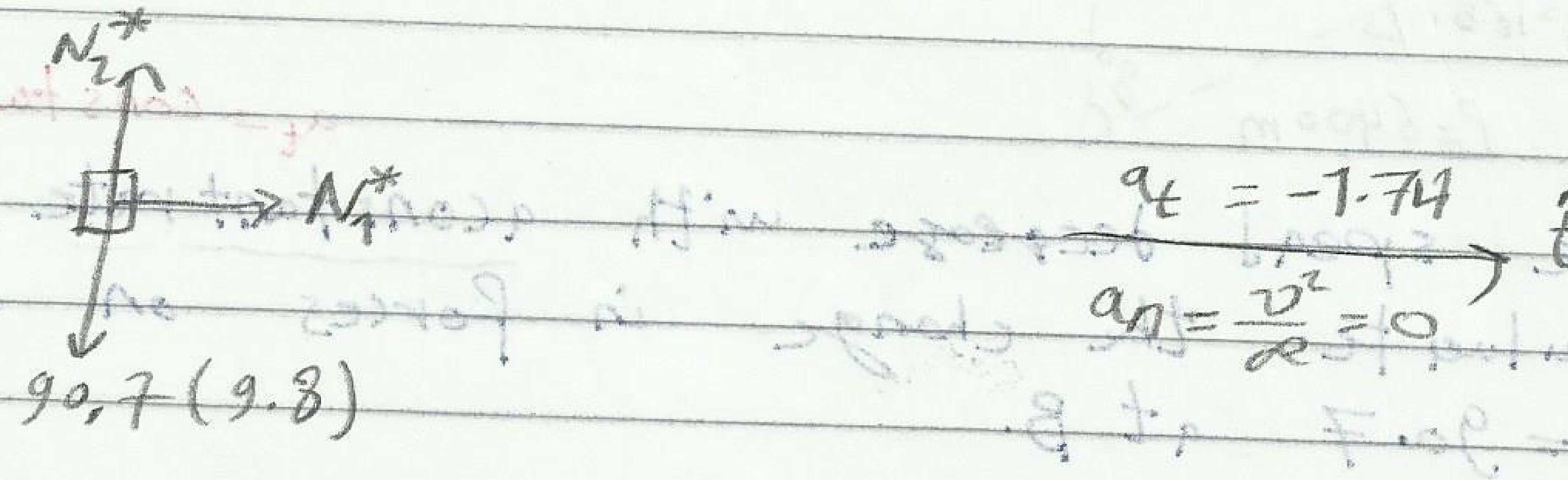


Acc. diag.

$\hat{t}$: $N_1 = 90.7(-1.74)$ هذا طبعاً يعني أنا ماخذها ثابت

$\hat{n}$: $(90.7)(9.8) - N_2 = (90.7)\left(\frac{(155)^2}{6400}\right)$

Just after B (line).



$\hat{t}$: $N_1^* = (90.7)(-1.74)$

$(90.7)(9.8) - N_2^* = m(0)$

$\Delta N_2 = N_2^* - N_2 = m\frac{v_B^2}{\rho} = \checkmark$

ال Normal زادت

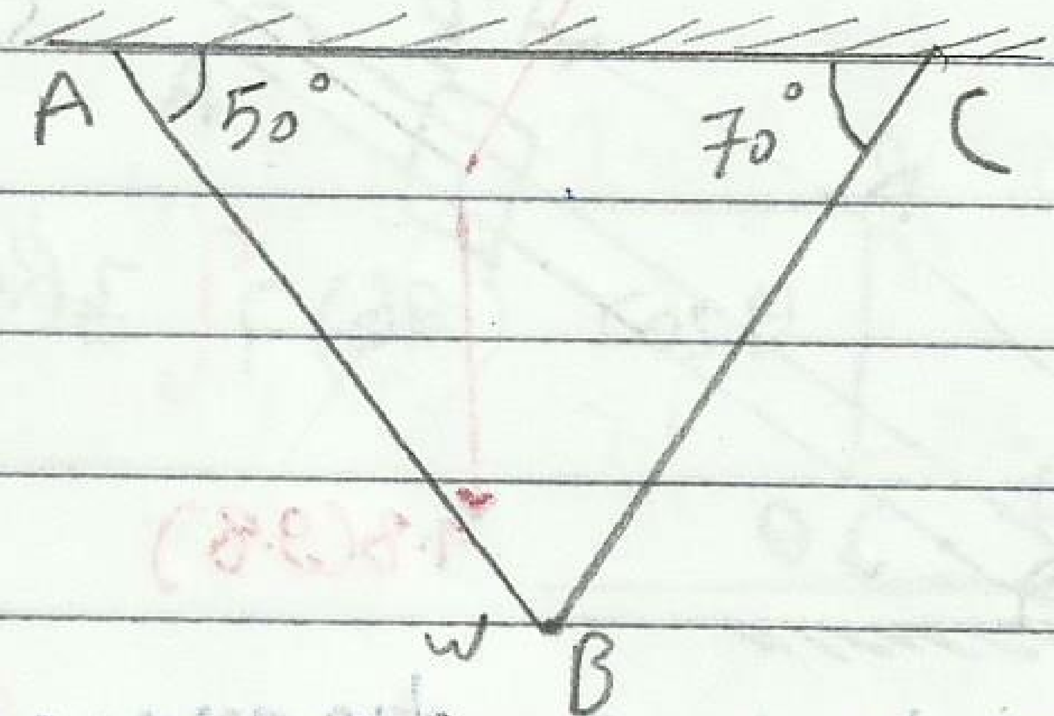
واحد 50 Kilo — واحد 70 Kilo ⇐

أكثر change ال بحس بال مين ↑

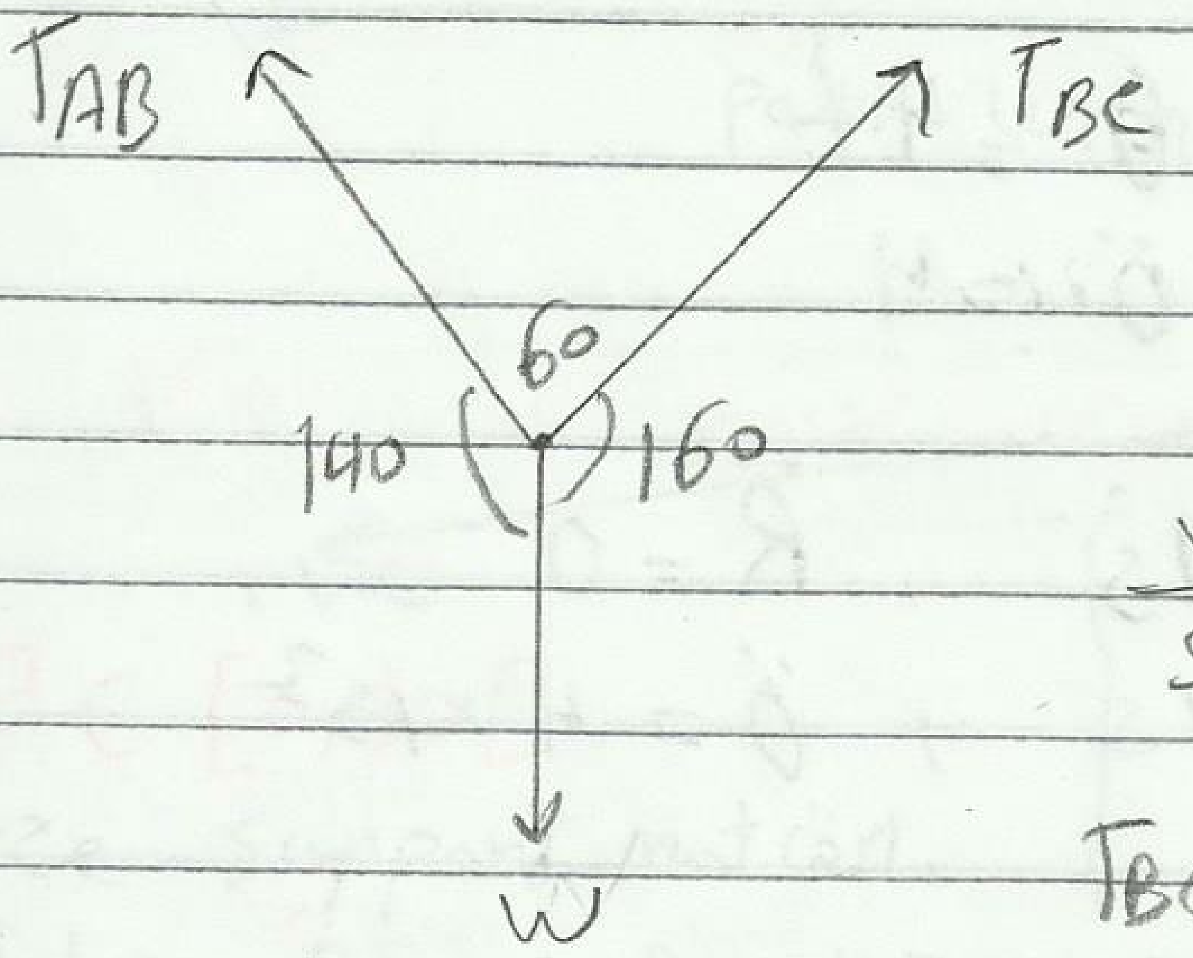
⇐ لو ال speed زادت ← يزيد التأثير

$\Delta N_2 \propto \frac{1}{\rho}$ ⇐ لو ρ زادت ← التأثير يقل

problem (12-44) page 713



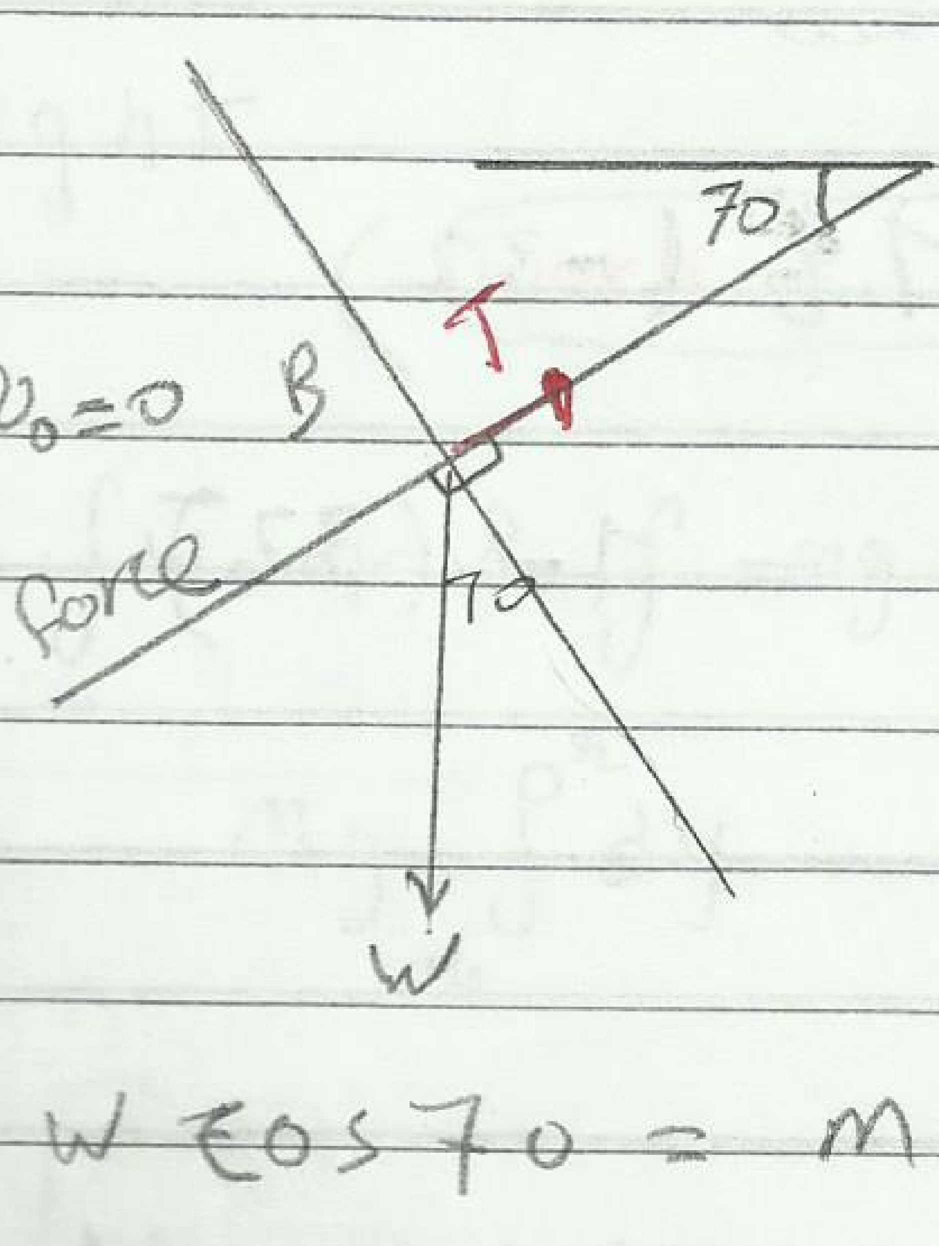
calculate the tension in cable BC
i) Before AB is cut.   ii) Just AB is cut.

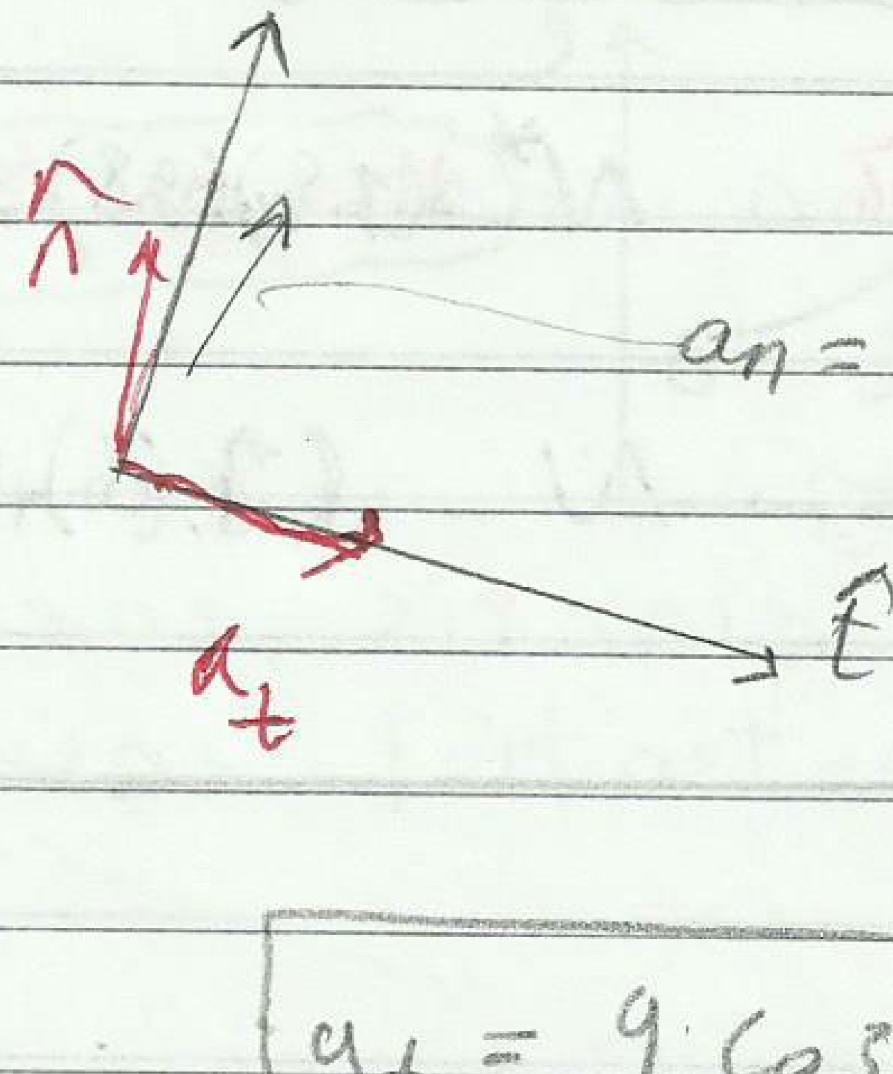


$$\frac{W}{\sin 60} = \frac{T_{BC}}{\sin 140} = \frac{T_{AB}}{\sin 160}$$

$$T_{BC} = W\left(\frac{\sin 140}{\sin 60}\right)$$

Just after AB is cut





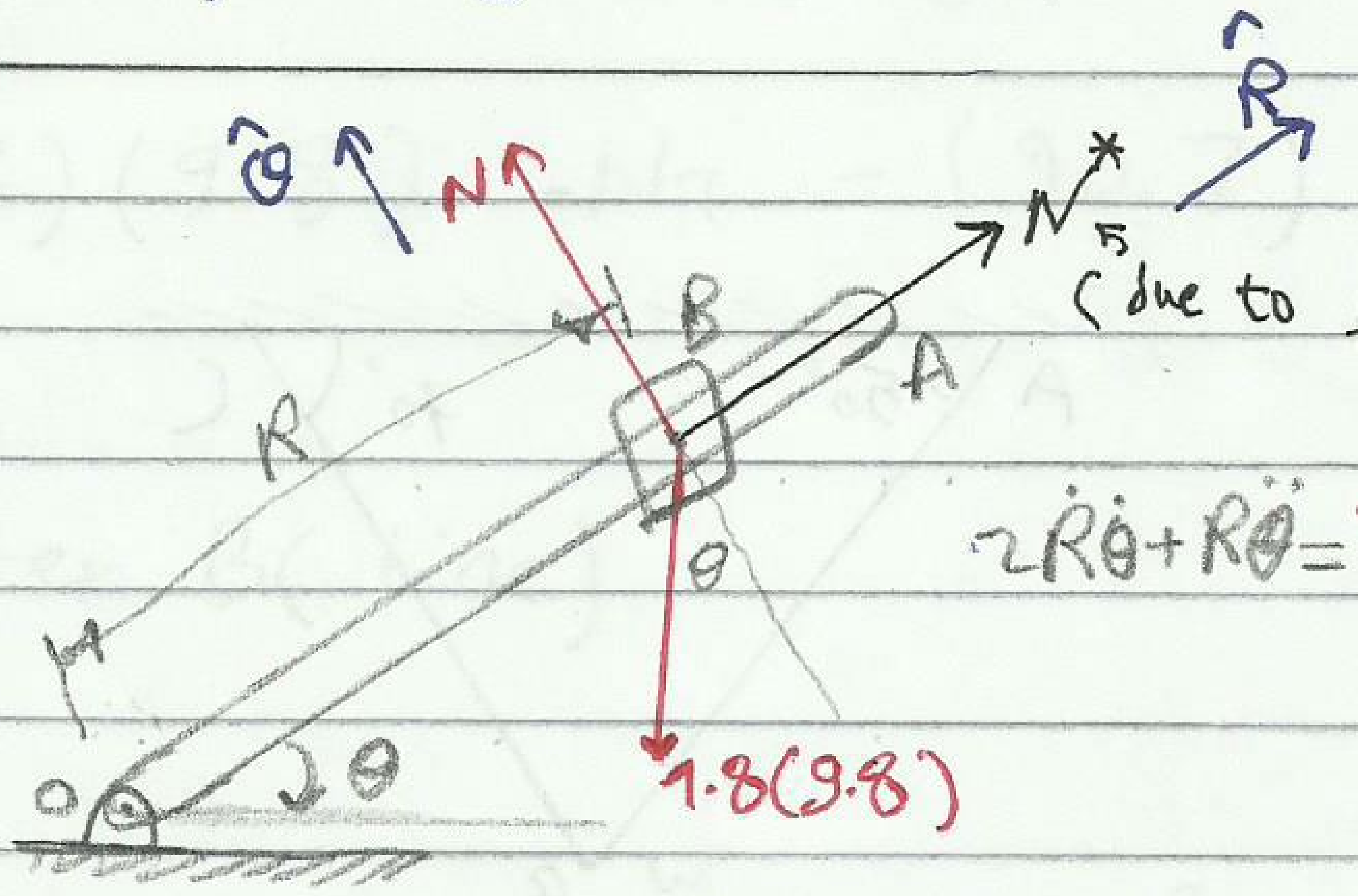
$$a_n = \frac{v^2}{\rho} = \frac{0^2}{\rho} = 0$$

$$W \cos 70 = m\, a_t \qquad \boxed{a_t = g \cos 70}$$

$$T^*_{BC} - W \sin 70 = m\left(\frac{v^2}{\rho}\right) \qquad T^*_{BC} = W \sin 70°$$

المفروض اقطع BC عشان ال particile يبدأ أسرع

problem (12.68) page 726



(due to friction)

$\boxed{m = 1.8\ Kg}$

$2\dot R\dot\theta + R\ddot\theta = a_\theta$

$a_R = \ddot R - R\dot\theta^2$

calculate the radial (friction force) & transverse reactions at t=0 & t=1s.

$R = 3t^2 - t^3$; $\theta = 2t^2$

$\dot R = 6t - 3t^2$ $\qquad \dot\theta = 4t$

$\ddot R = 6 - 6t$ $\qquad \ddot\theta = 4$

at $t = 1s$; $R = 2m$, $\dot R = 3 m/s$, $\ddot R = 0$

$\theta = 2rad$, $\dot\theta = 4 r/s$, $\ddot\theta = 4 r/s^2$

$a_R = -32\ m/s^2$, $a_\theta = 24 + 8 = 32\ m/s^2$

Equations of motion:- $\frac{2\pi}{180}$ [illegible]

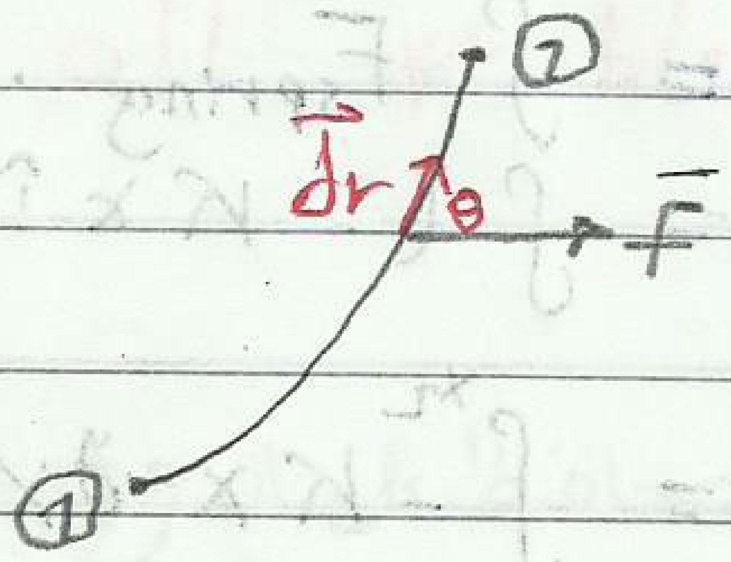
$\hat R$ $\qquad N^* - (1.8)(9.8)\sin\theta = 1.8(-32)$ (2rad)

$\hat\theta$ $\qquad N - (1.8)(9.8)\cos\theta = 1.8(32)$

chapter (13)

work - Energy (force - ~~...~~ velocity - distance method)

(1) work



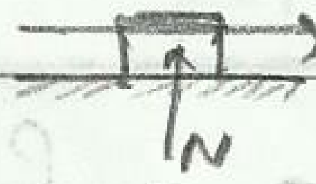
$$\boxed{U_{1 \to 2} = \int du = \int \bar{F} \cdot \bar{dr} = \int F (dr) \cos\theta}$$

special case: F = constant , θ = constant.

$$\boxed{U_{1 \to 2} = F\, d \cos\theta}$$

path lenth

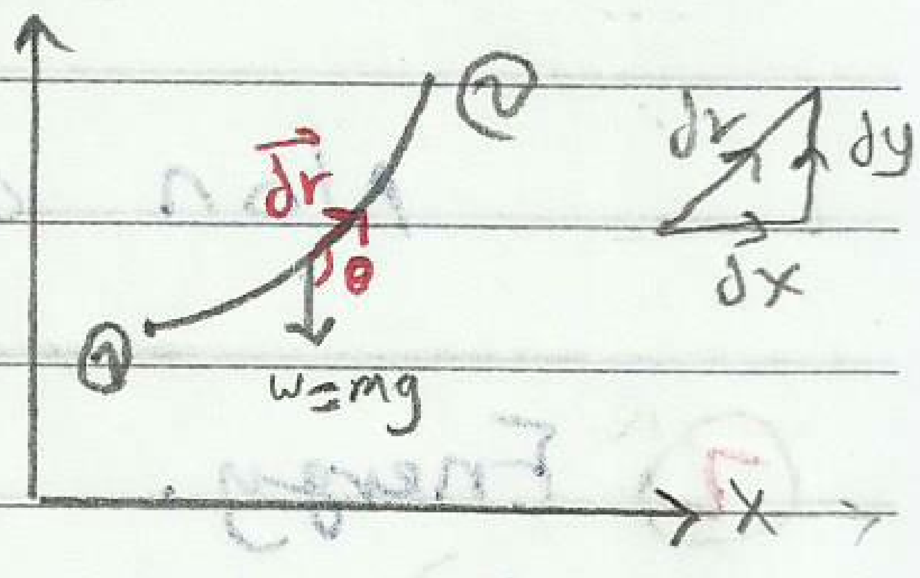
$U_{1 \to 2}$

+ve	zero	-ve
$\theta \in [0, 90[$	$\theta = 90°$	$\theta \in]90, 180[$
force support motion.		force resist motion

- Work of common forces;

1. weight

work is independent on path



$$U_{1 \to 2} = \int \vec{w} \cdot dr = \int -mg\, \hat{j} \cdot (dx\, \hat{i} + dy\, \hat{j})$$

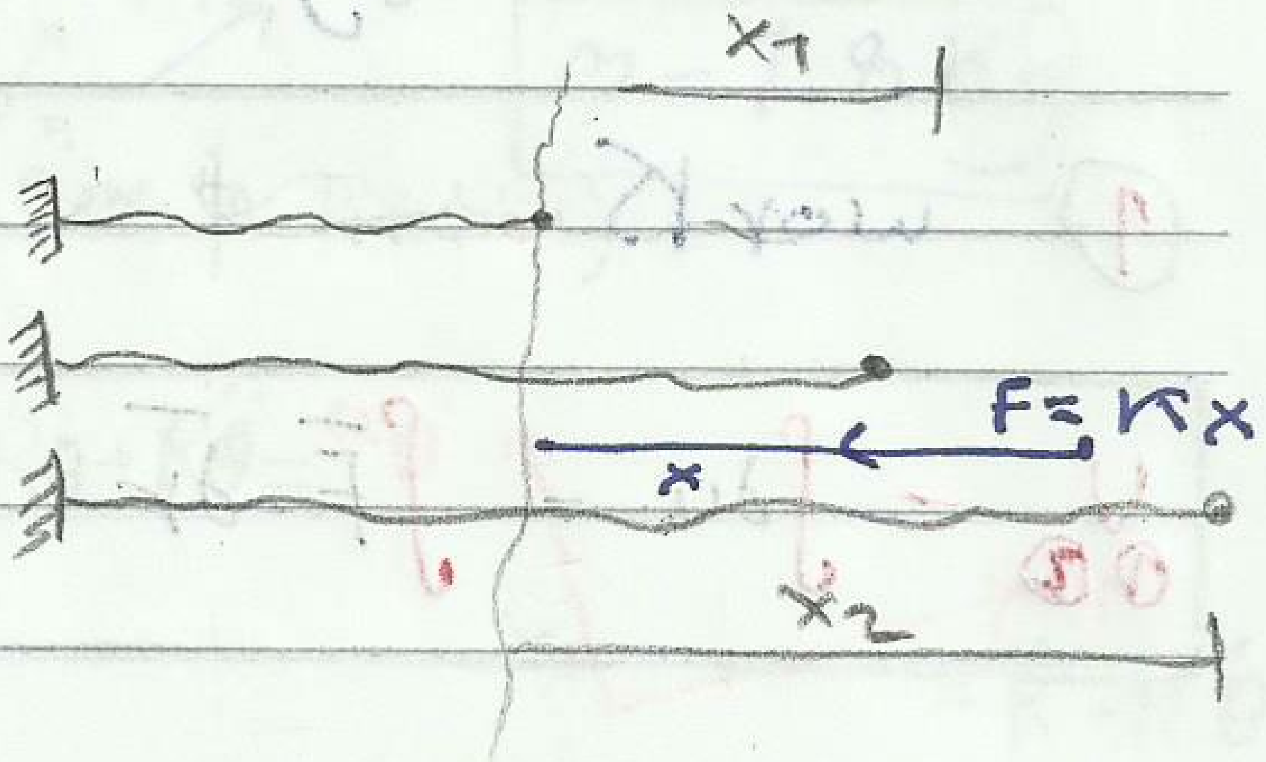
$$= -mg \int_{y_1}^{y_2} dy$$

$$\boxed{U_{1 \to 2} = mg y_1 - mg y_2} = mg\, \Delta y$$

لو نازل مع ال gravity ↓ +y ، وطالع ↑ يبقى -y

2. spring.

$$U_{1\to2} = \int \vec{F}_{spring} \cdot \vec{dr}$$
$$= \int (-Kx\,\hat{i}) \cdot (dx\,\hat{i} + dy\,\hat{j})$$
$$= \int_{x_1}^{x_2} -Kx\,dx = -K \frac{x^2}{2}\Big|_{x_1}^{x_2}$$



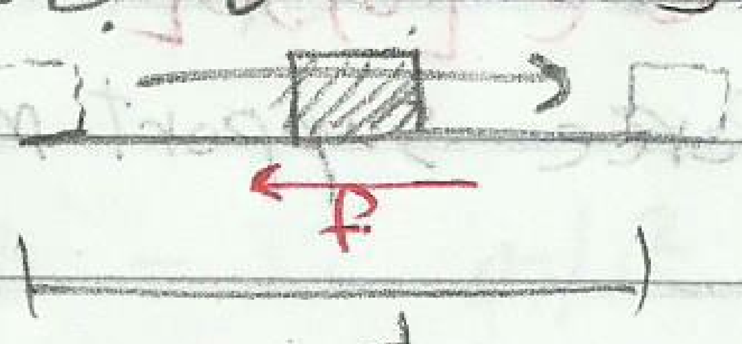
$$\boxed{U_{1\to2} = \frac{1}{2}Kx_1^2 - \frac{1}{2}Kx_2^2}$$

⇐ لو $x_2 > x_1$ يبقى +ve .

3. friction

work is dependent on path length

الـ fric عكس إتجاه الحركة.



$$\boxed{U_{1\to2} = f\,d\cos 180 = -f\,d}$$

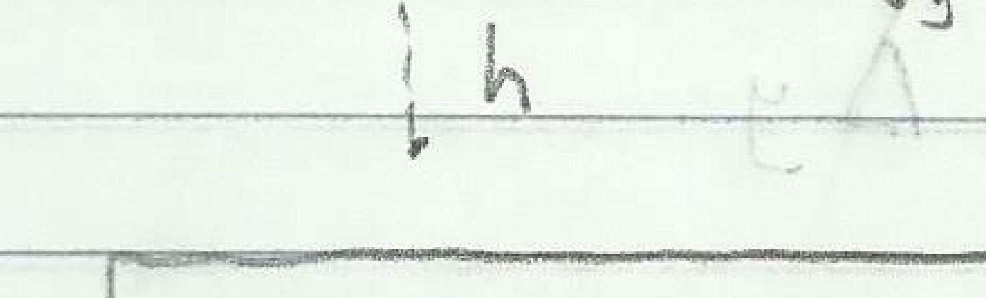
لازم يطلع -ve لأنه ...

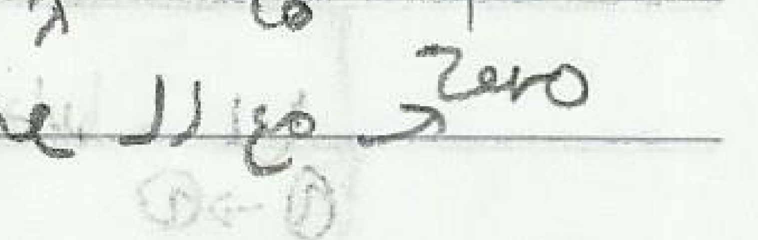
Non conservative.

② Energy.

i) kinetic energy (T) $\boxed{T = \frac{1}{2} m v^2}$

ii) potential energy (V)





unstretched line مع الـ zero

$\boxed{V_g = \pm m g h}$ gravity

$\boxed{V_s = \frac{1}{2} K x^2}$ spring +ve دائماً

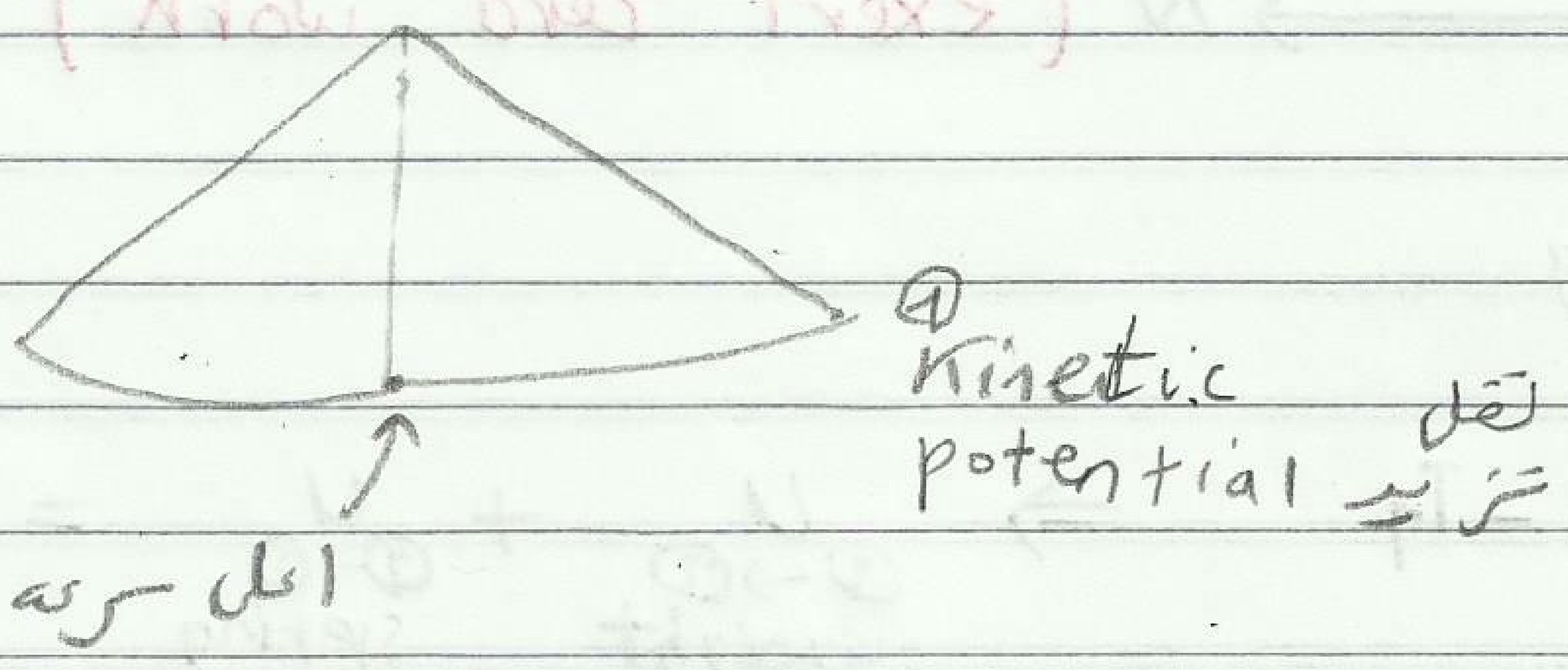
③ work - Energy relation.

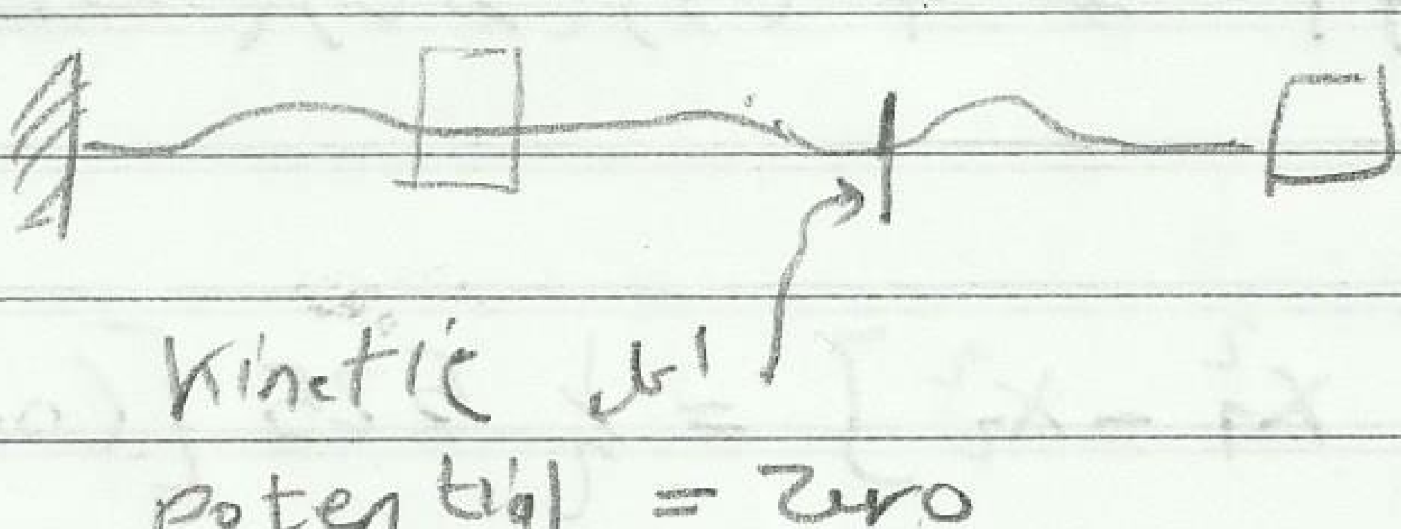
$$\boxed{U_{1\to 2} = T_2 - T_1} \Longrightarrow \text{for all problems.}$$

for conservative بيحافظ على الـ energy

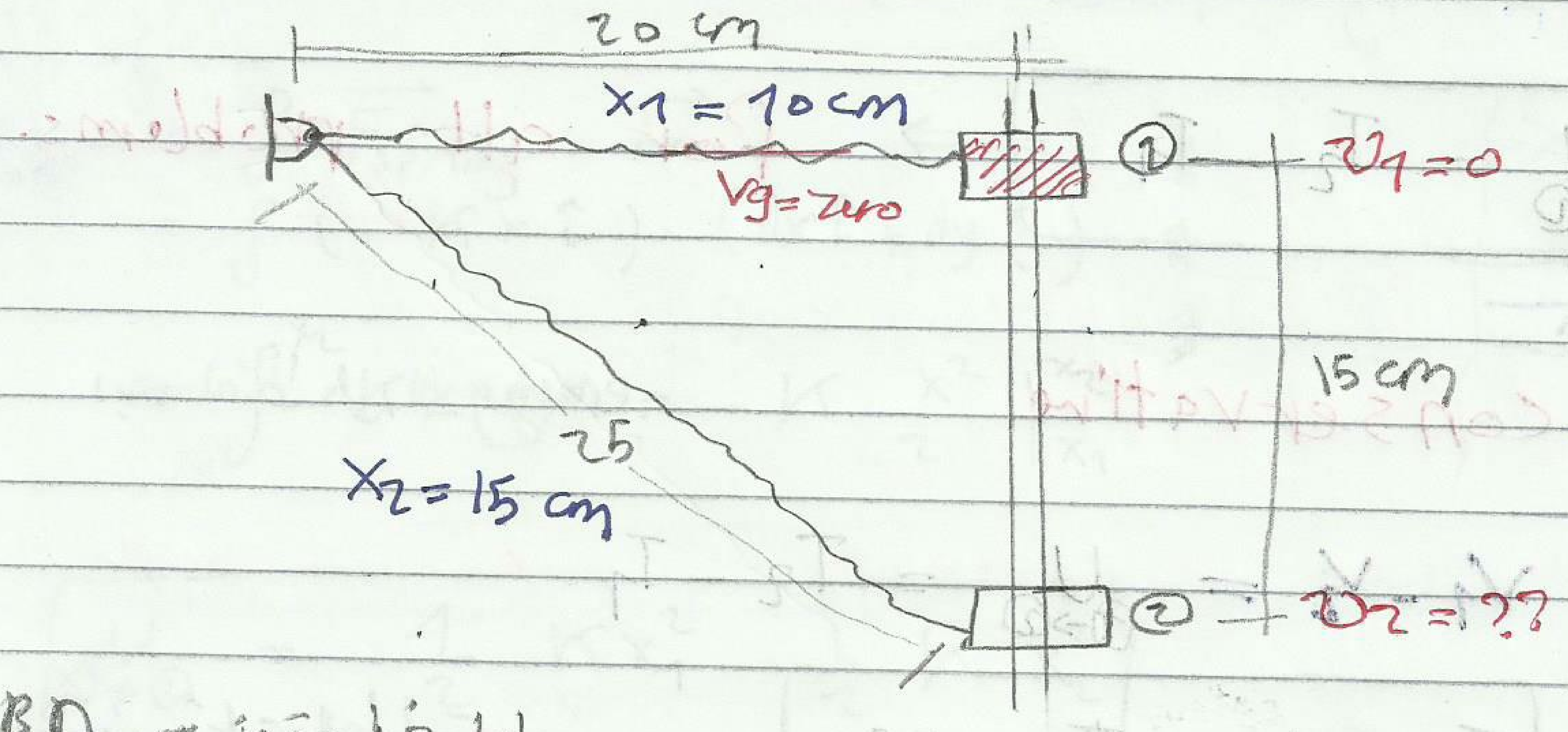
$$V_1 - V_2 = U_{1\to 2} = T_2 - T_1$$

$$\boxed{T_1 + V_1 = T_2 + V_2}$$ قانون ثبات الطاقة





Ex (13-6) page 787



m = 9 kg

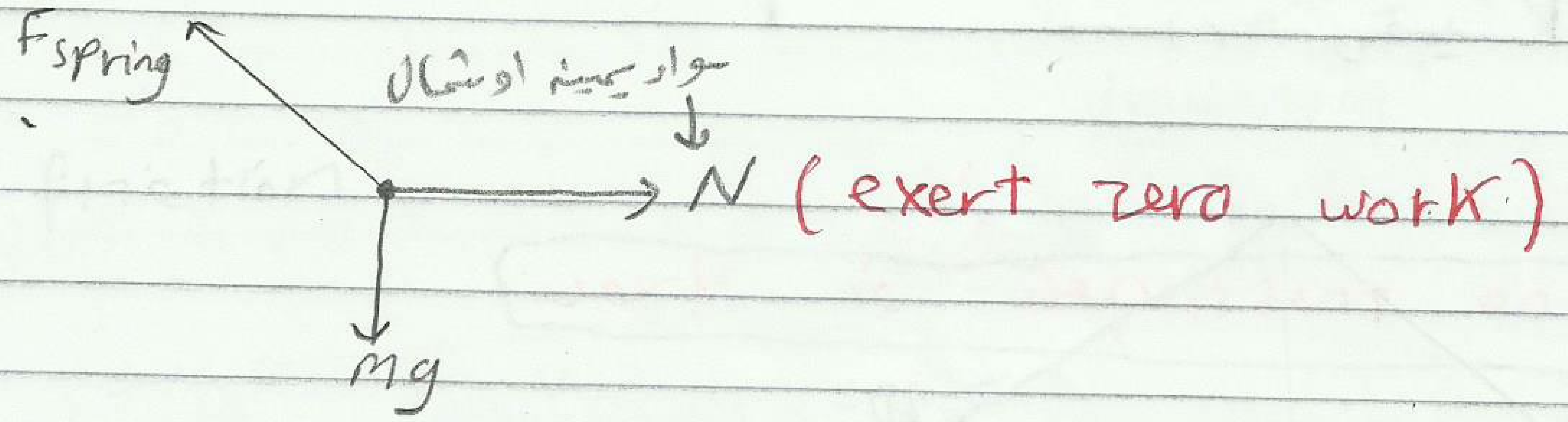
$l_0 = 10$ cm

$K_{spring} = 525$ N/m

moves without friction

$\boxed{\text{Calculate } v_2}$

FBD اول خطوة نرسم



$$\underset{①\to②}{U} = T_2 - T_1 \Rightarrow \underset{\substack{①\to②\\ \text{weight}}}{U} + \underset{\substack{①\to②\\ \text{spring}}}{U} = T_2 - T_1$$

نازل لتحت ↓

$$\underset{\substack{①\to②\\ \text{weight}}}{U} = +mg\,|\Delta y| = +(9)(9.8)(0.15) = + \text{---}\ J$$

$$\underset{\substack{①\to②\\ \text{spring}}}{U} = \frac{1}{2} K \left[x_1^2 - x_2^2 \right] = \frac{1}{2}\, 525 \left[(0.1)^2 - (0.15)^2 \right] =$$

$$T_1 = \frac{1}{2} m v_1^2 = 0$$

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}\, (9)\, v_2^2$$

Another solution

$$\boxed{T_1 + V_1 = T_2 + V_2}$$

لازم weight او spring بس
عشان استخدم القانون

$$V_1 = V_{1g} + V_{1s} = mg(0) + \frac{1}{2}k(0.1)^2$$

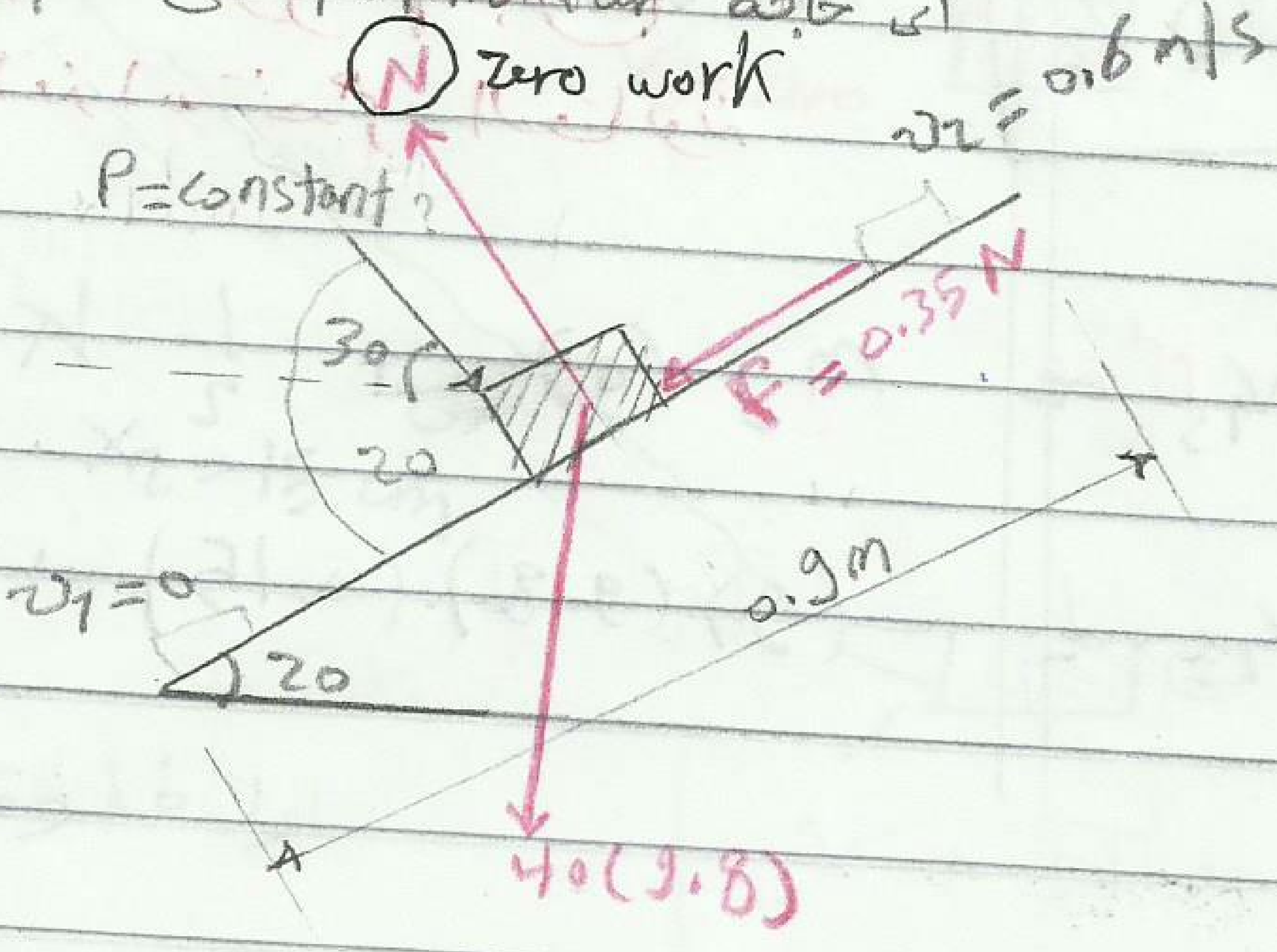
$$V_2 = V_{2g} + V_{2s} = -(9)(9.8)(0.15) + \frac{1}{2}k(0.15)^2$$

/07/2013

problem (13.9) page 772 ⇓ مسألة مهمة أوي

By using 3 methods.

أي حاجة perpendicular على ال displacement ماعملهاش ورك



$m = 40$ Kg

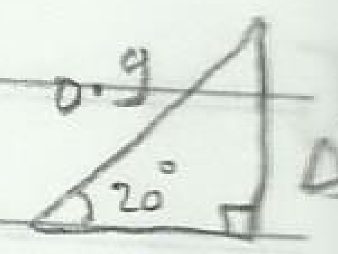
$M_K = 0.35$

calculate: P such that $v_1 = 0$, $v_2 = 0.6$ m/s in a distance 0.9 m.

$$\underset{\text{all forces}}{U_{1\to 2}} = T_2 - T_1$$

$$\underset{\text{weight}}{U_{1\to 2}} + \underset{\text{force } p}{U_{1\to 2}} + \underset{\text{friction}}{U_{1\to 2}} = T_2 - T_1$$

$$\underset{\text{weight}}{U_{1\to 2}} = -mg\,|\Delta y| = -40(9.8)\left[0.9 \sin 20\right]$$



مع ال plane

$$\underset{\text{force } p}{U_{1\to 2}} = Fd\cos\theta = P(0.9)\cos 50$$

ال angle بين ال force و displacement
داخلين على نقطة واحدة

$$\underset{\text{friction}}{U_{1\to 2}} = -f\,(\text{Path length}) = -(0.35N)(0.9) = -\left[0.35\left(P\sin 50 + 40(9.8)\cos 20\right)(0.9)\right]$$

N ↑↓ $N - P\sin(50) - w\cos 20 = m(0)$

$$T_1 = \frac{1}{2} m v_1^2 = 0$$

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(40)(0.6)^2$$

← لو مفيش friction ← الطاقة ثابتة ← مش محتاج P كبيرة.

← μ_k كبرت ← P المفروض تكبر.

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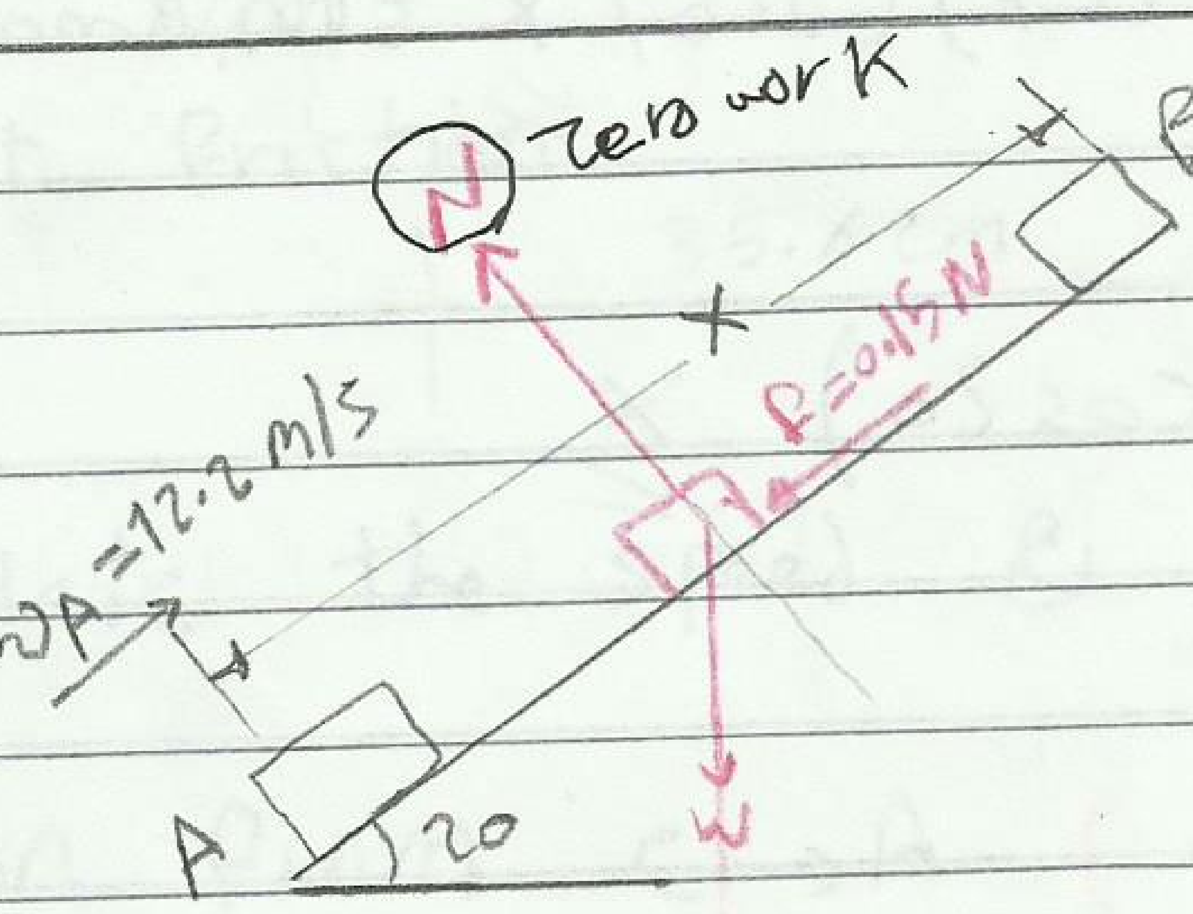
problem (13.6) page 771



$M = 22.7$ Kg

$\mu_k = 0.15$

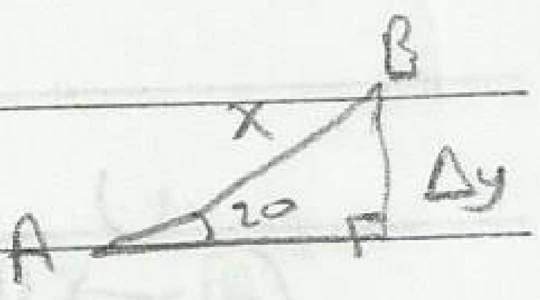
calculate: max travel distance x $v_B = 0$ (اقصى مسافة يطلعها)

, $(v_A)_{return}$, the enegy dispated فقدة due to friction.

$$N - w \sin 20 = m(0)$$

$$\underset{\substack{1 \to 2 \\ \text{all forces}}}{U} = T_2 - T_1 \Rightarrow \underset{\substack{A \to B \\ \text{weight}}}{U} + \underset{\substack{A \to B \\ \text{friction}}}{U} = T_B - T_A$$

$$\underset{A \to B}{U} = -mg|\Delta y| = -(22.7)(9.8)[x \sin 20]$$



$$\underset{\substack{A \to B \\ \text{friction}}}{U} = -[0.15(22.7(9.8))\cos 20°]\, x$$

$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2}(22.7)(12.2)^2$$

$$T_B = \frac{1}{2} m v_B^2 = 0 \qquad \boxed{x = \checkmark \, m}$$

② motion from $B \to A$

$$U_{\substack{B \to A \\ weight}} + U_{B \to A} = T_A - T_B$$

$$U_{\substack{B \to A \\ weight}} = + mg(\Delta y) = +(22.7)(9.8)\, x \sin 20$$

zero work

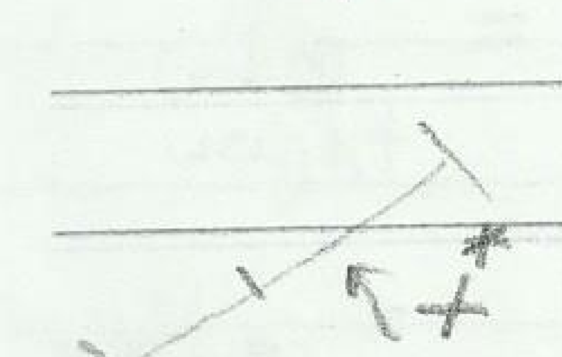
$$U_{\substack{B \to A \\ friction}} = -0.15\,(22.7\,(9.8)\cos 20)\; x$$

هوا هوا

$$T_B = -\frac{1}{2} m v_B^2 = 0$$

$$T_{A(return)} = \frac{1}{2}(22.7)\, v_{A(return)}^2$$

$v_A = v_A$ ... friction لكن لو مفيش لازم يقل $v_{A\,(return)}$



لو طلعت v_A بـ (-) معناها انو ما فيش يوصل لـ A

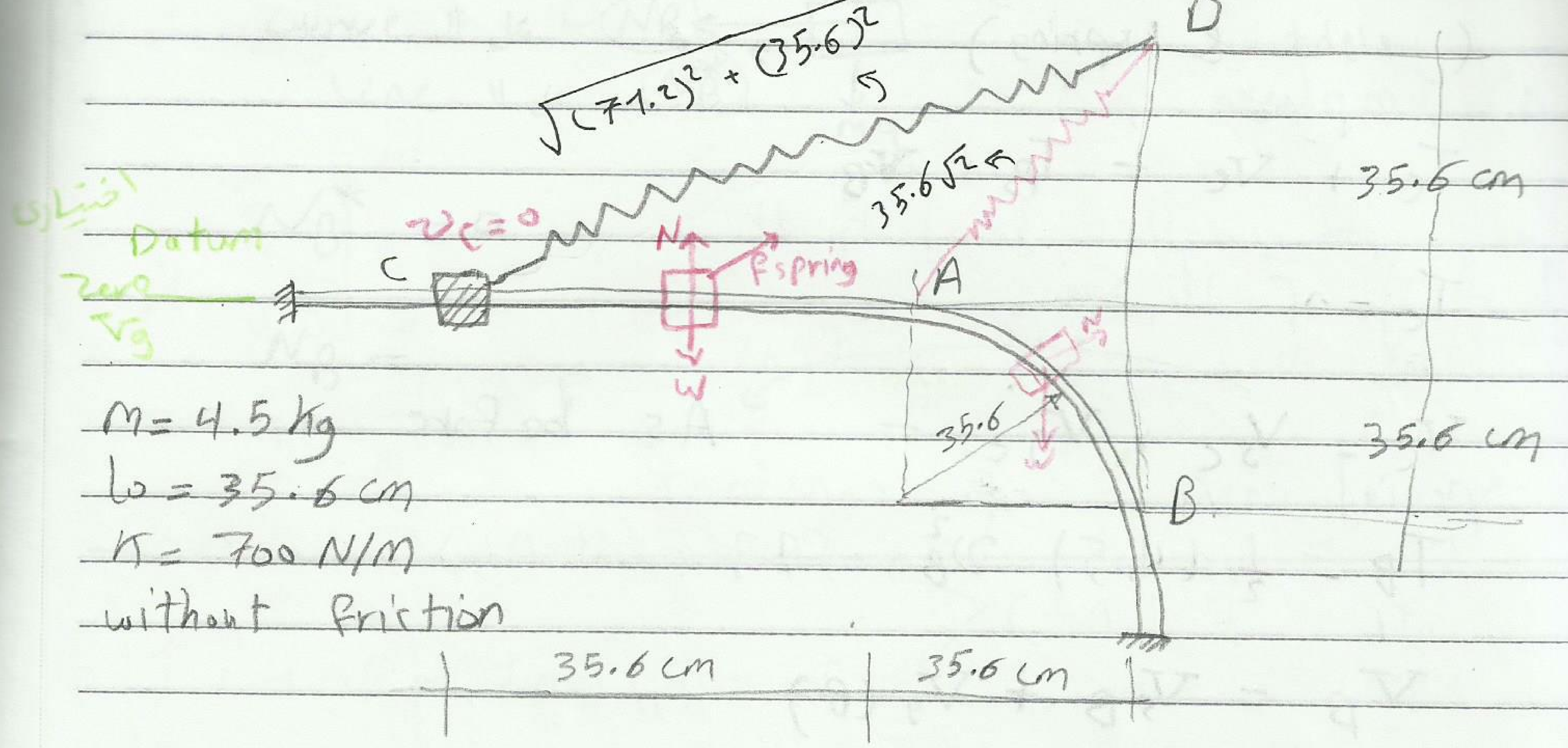
③ $$\Delta T = (T_A)_{return} - (T_A)_{initial}$$
$$= \frac{1}{2}(22.7)\, v_{A(return)}^2 - \frac{1}{2}(22.7)(12.7)^2$$

$$U_{\substack{A \to A \\ up \quad down}} = T_{A\,return} - T_{A\,up}$$

$$-2\,U_{friction} = \Delta t$$

Vwork [illegible] Vwork [illegible]

Problem (13.66) page 795



$m = 4.5$ kg
$l_0 = 35.6$ cm
$K = 700$ N/M
without friction

calculate the speed & Normal reaction at i) A, ii) B

motion from $C \to A$ (driving force spring)

$$T_C + V_C = T_A + V_A \qquad \text{or} \quad \left(U_{C \to A} = T_A - T_C \right)$$

$$T_C = \frac{1}{2} m v_C^2 = 0 \quad , \quad T_A = \frac{1}{2}(4.5)\, v_A^2 = ??$$

$$V_C = \frac{1}{2} K x_C^2 = \frac{1}{2}(700)\left[\frac{\sqrt{(71.2)^2+(35.6)^2} - 35.6}{100}\right]^2 \leftarrow CD - l_0$$

لتحويل من cm ← m

$$V_A = \frac{1}{2} K x_A^2 = \frac{1}{2}(700)\left[\frac{35.6\sqrt{2} - 35.6}{100}\right]^2 \quad AD - l_0$$

$$v_A = \checkmark$$

$$U_{C \to A} = \frac{1}{2} K x_C^2 - \frac{1}{2} K x_A^2 \rightarrow$$ لو مستخدم بالقانون الثاني

motion from $C \to B$ بدل

من $A \to B$

(weight & spring)

$$T_C + V_C = T_B + V_B$$

$$T_C = 0$$

$$V_C = V_{s_C} + V_{g_C} = \text{As before}$$

$$T_B = \frac{1}{2}(4.5)\, v_B^2 = ??$$

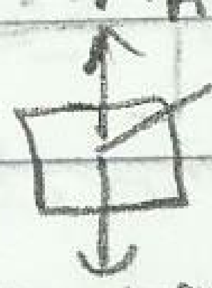
$$V_B = V_{s_B} + V_g(B)$$

$$= \frac{1}{2}(700)\left[\frac{71.2 - 35.6}{100}\right]^2 - 4.5(9.8)(0.356)$$

$v_B = \checkmark$

Normal reaction at A

Before



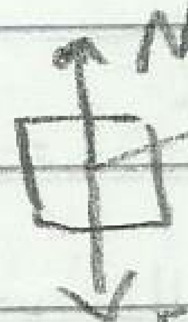
N_A $\rightarrow F_{spring} = Kx = 700\left[\frac{35.6\sqrt{2} - 35.6}{2}\right]$

$W = mg = 4.5(9.8)$

$a_n = 0$ $\quad a_t = ?$

$$4.5(9.8) - N_A - F_{spring}\sin 45^\circ = m(0)$$

after



N_A^* $\rightarrow F_{spring}$

$4.5(9.8)$

$a_t = ?$

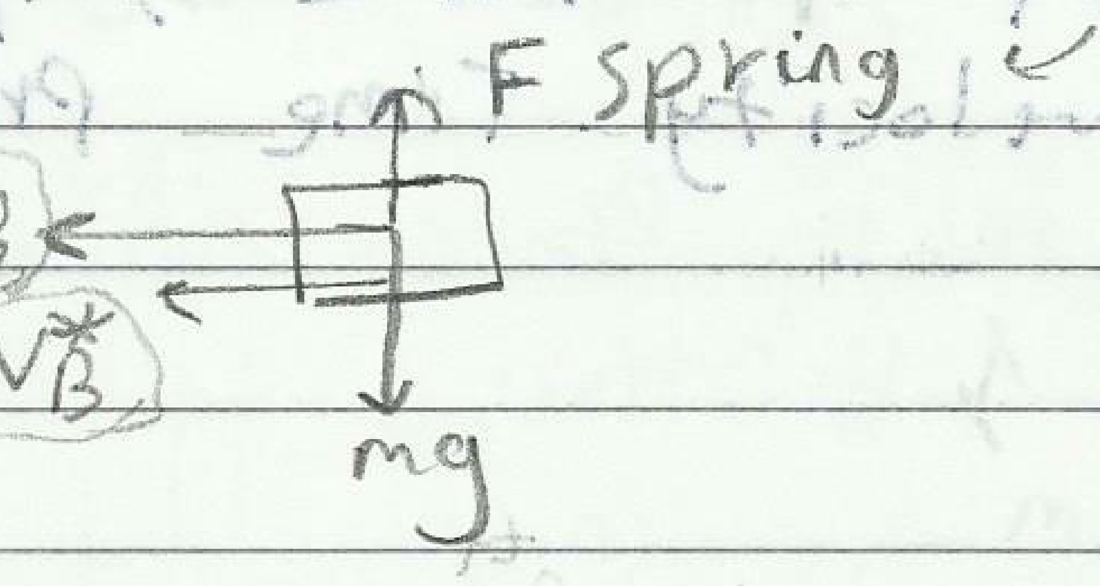
$a_n = \frac{v_A^2}{0.356}$

$$(4.5)(9.8) - N_A^* - F_{spring}\sin 45 = 4.5\left(\frac{v_A^2}{0.356}\right)$$

$N_A^* = \checkmark$

لو طلب الـ change بيتغير

Normal reaction at B



curve عل ال ← N_B

line عل ال ← N_B^*

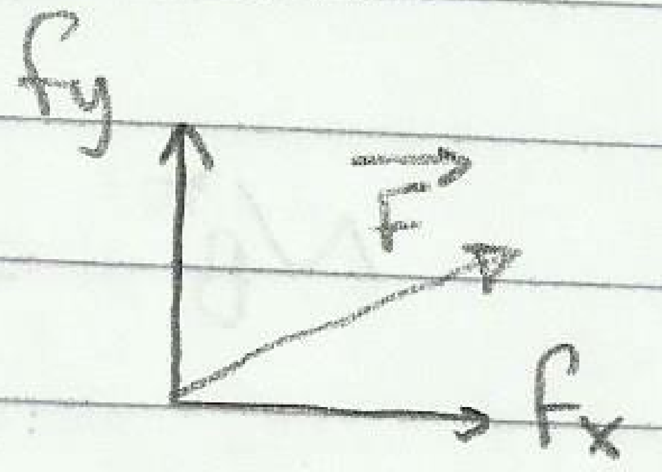
$$N_B^* = 0$$

$$N_B = \frac{\quad}{\rho}$$

Kinetics of a particle: Impluse & momentum principle
(Force - velocity - Time principle)

1) Impluse I

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt \;\begin{cases} I_x = \int_{t_1}^{t_2} F_x\, dt \\ I_y = \int_{t_1}^{t_2} F_y\, dt \end{cases}$$

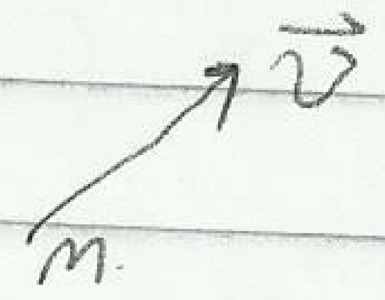


Special case :- $\vec{F}$ = constant

$$\vec{I} = \vec{F}\,(\Delta t) \;\begin{cases} I_x = F_x\,(\Delta t) \\ I_y = F_y\,(\Delta t) \end{cases}$$

2) Momentum كمية التحرك ($\vec{P}$ or $\vec{L}$)

$$\vec{P} = m\,\vec{v} \;\begin{cases} P_x = m\, v_x \\ P_y = m\, v_y \end{cases}$$



3) Impluse - Momentum relationship.

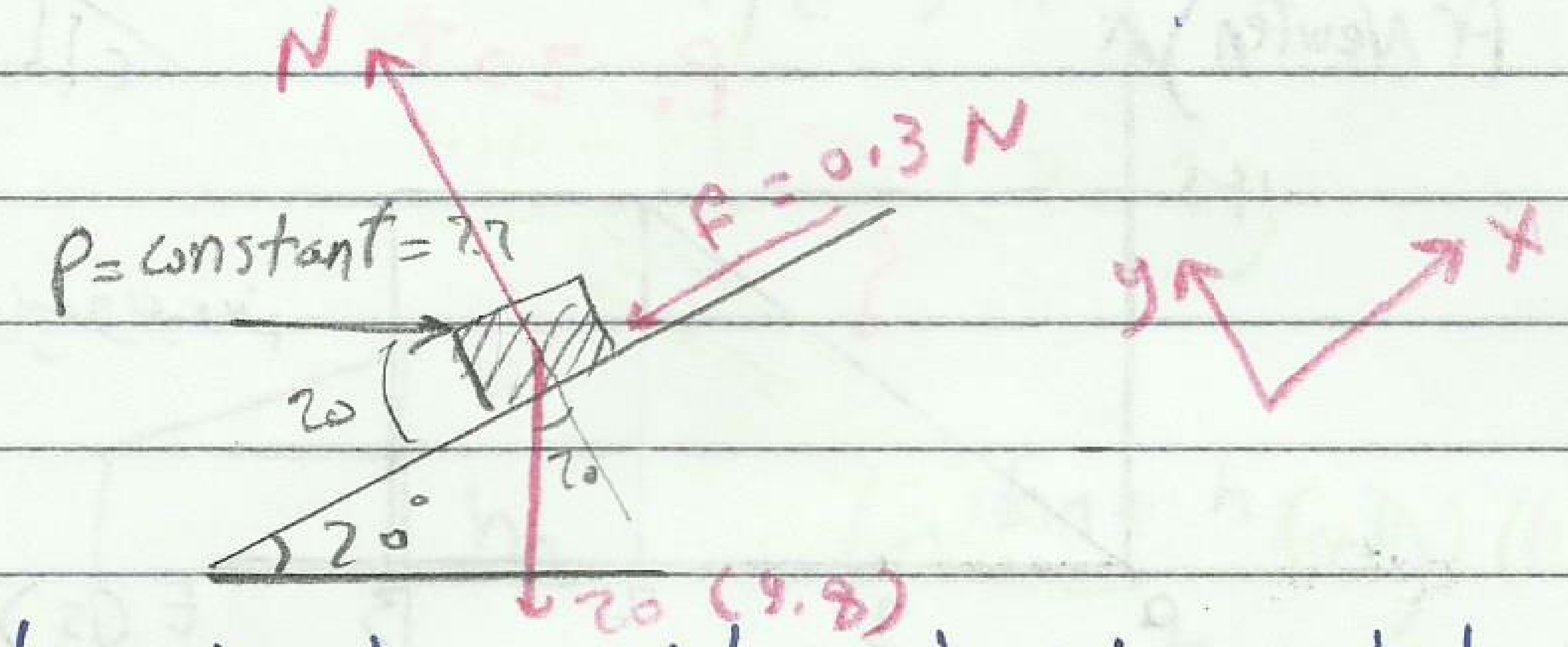
$$\vec{F} = m\,\vec{a} \Rightarrow \vec{F} = m\,\frac{d\vec{v}}{dt} \Rightarrow \int_{t_1}^{t_2} \vec{F}\, dt = m \int_{\vec{v}_1}^{\vec{v}_2} dv$$ mass constant

$$\boxed{\int_{t_1}^{t_2} \vec{F}\, dt = m\,\vec{v}_2 - m\,\vec{v}_1} \longrightarrow \boxed{\vec{I} = \Delta \vec{P}}$$

$$\int F_x\, dt = m\,(v_2)_x - m\,(v_1)_x$$
$$\int F_y\, dt = m\,(v_2)_y - m\,(v_1)_y$$

problem (13-126) page 814

By using 3 methods



$m = 20\ Kg$

$\mu_k = 0.3$

$v_1 = 0$

$v_2 = 15\ m/s$ } time 6 s

calculate the horizontal force P = ??

$\boxed{I_x = \Delta P_x}$ $\int_0^6 (P\cos 20 - 20(9.8)\sin 20 - 0.3N)\,dt = 20(v_2)_x - 20(0)$

($t_1 = 0$ طالما مقالش (دائماً)

بيتحرك على x

$$\underbrace{(P\cos 20 - 20(9.8)\sin 20 - 0.3N)}_{\Sigma F_x}\underbrace{(6)}_{\Delta t} = 20(15) - 20(0) \quad — ①$$

$\boxed{I_y = \Delta P_y}$ $\int_0^6 (N - 20(9.8)\cos 20 - P\sin 20)\,dt = 20(v_2)_y - 20(0)$

$(N - 20(9.8)\cos 20 - P\sin 20)(6) = 20(0) - 20(0)$ — ②

مبيتحركش على y

Another solution (Force - Acceleration method)

$F_x = m a_x$: $P\cos 20 - 20(9.8)\sin 20 - 0.3N = 20a$

$F_y = m a_y$: $N - 20(9.8)\cos 20 - P\sin 20 = 20(0)$

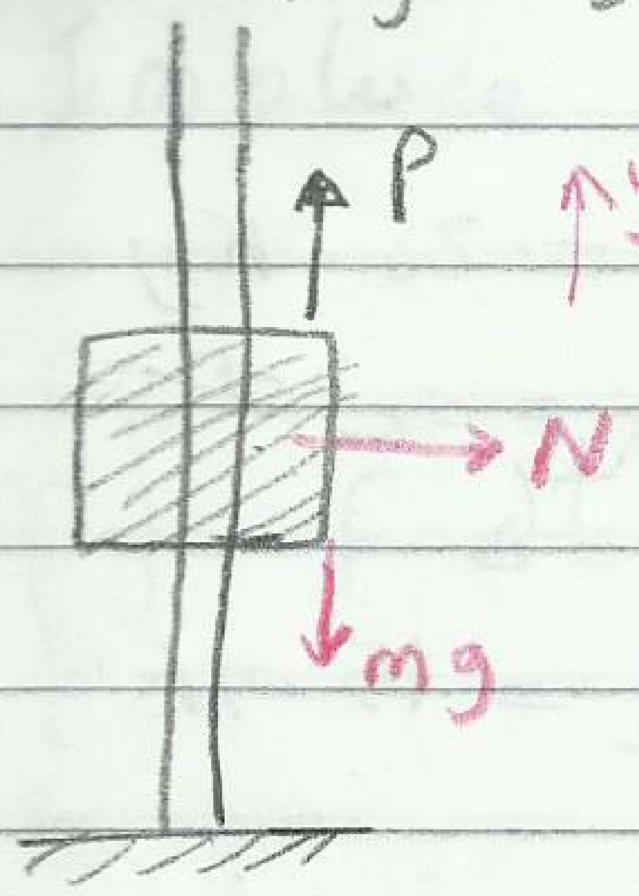
$v_2 = v_1 + at$

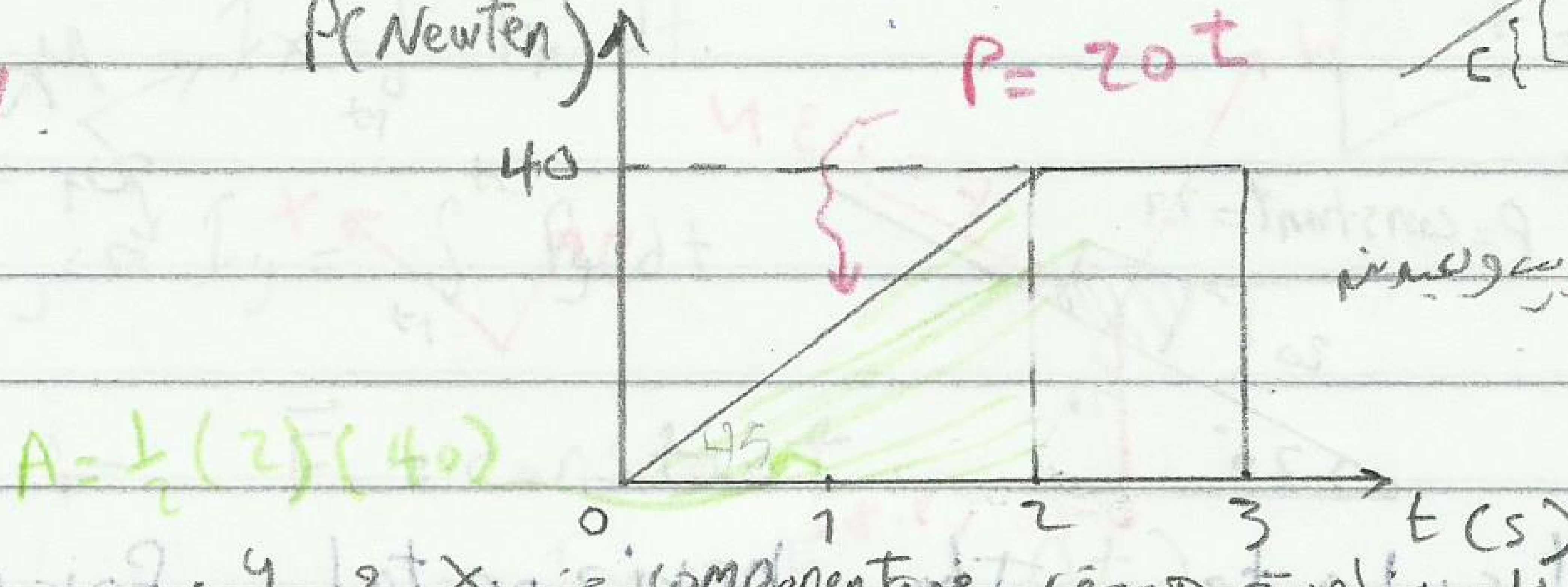
$15 = 0 + a(6) \Rightarrow \boxed{a = 2.5}$

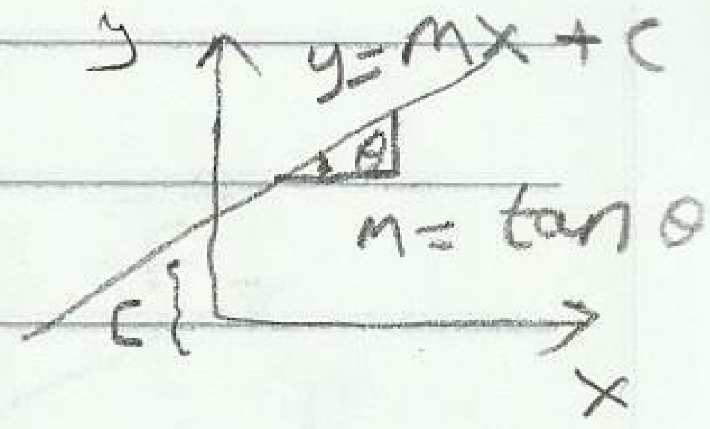
problem (13-136) page: 815 حل مجموعة اولى

m = 2 kg starts from rest , calculate: v_{2s} ; v_{3s}

friction مفيش







لو P مائلة بزاوية هيبقى في component في x و y.

$\boxed{I_x = \Delta P_x}$ $\int N\,dt = m(v_2)_x - m(v_1)_x$

$N(\Delta t) = 2(0) - 2(0)$ $\boxed{N = 0}$

$\boxed{I_y = \Delta P_y}$ $\int_0^2 (P - 2(9.8))\,dt = m(v_{2s})_y - m(v_0)_y$

at $t = 2s$ constant الـ P مش

$\left[\int_0^2 20t\,dt\right] - 2(9.8)(2-0) = 2\,v_{2s}$

السرعة في اتجاه y

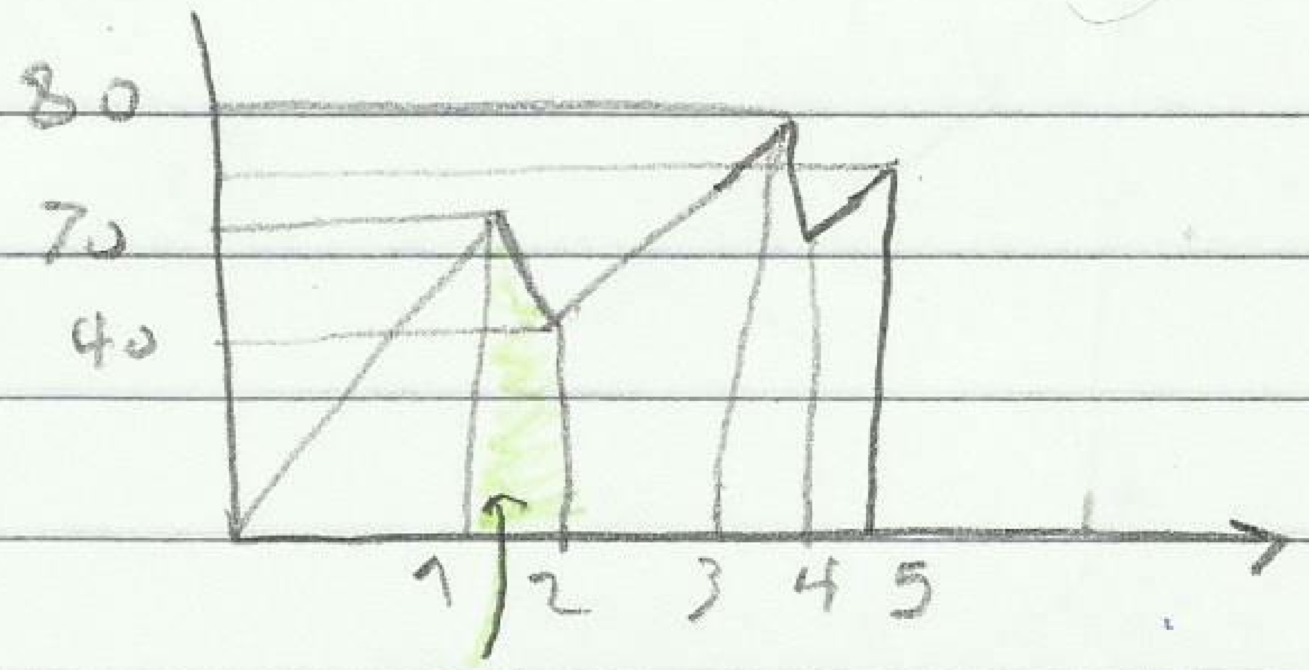
$\left[10t^2\right]_0^2 - 2(9.8)(2) = 2\,v_{2s}$

$\therefore v_{2s} = \dfrac{40 - 2(9.8)(2)}{2}$

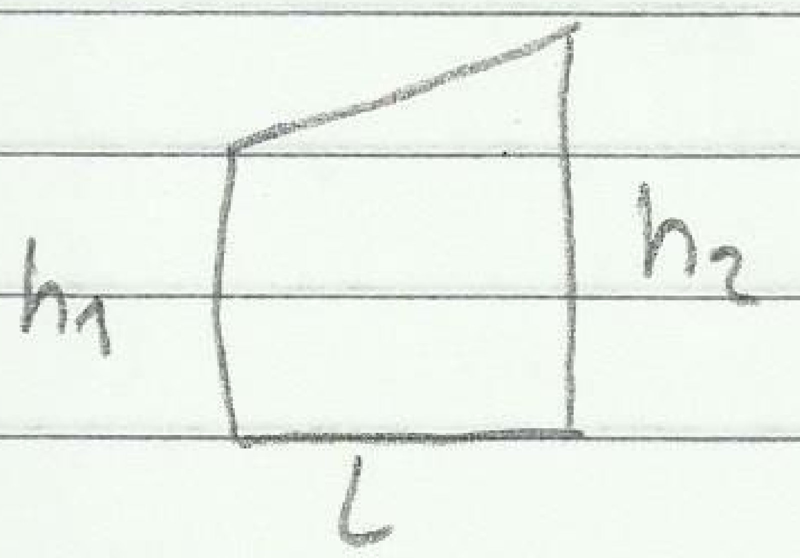
at $t = 3s$ $\int_0^3 [P - 2(9.8)]\,dt = m v_{3s} - m v_0$

$\int_0^2 20t\,dt + \int_2^3 40\,dt - 2(9.8)(3) = 2 v_{3s} - 2(0)$

$40 + 40(1) - 2(9.8)(3) = 2\,v_{3s}$



$\frac{40+70}{2} \times (2-1)$



$\left(\frac{h_1+h_2}{2}\right) L$

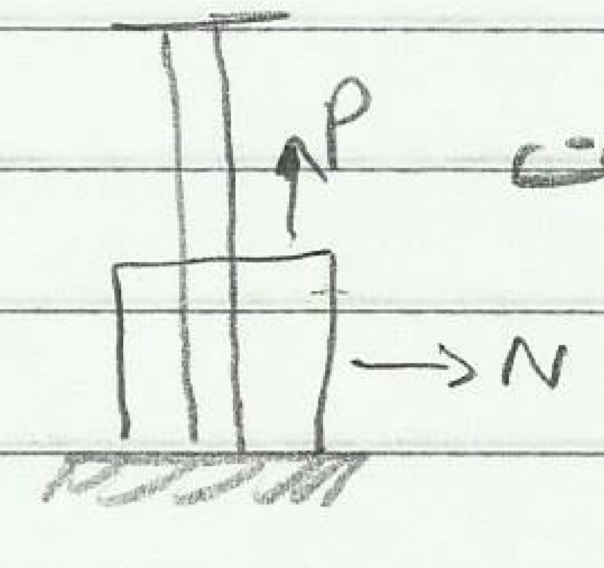
لما يطلب مثلا الوقت الى يكون عنده الجسم ساكن لحظياً

$\int_0^2 20t\,dt + \int_2^3 40\,dt - (2)(9.8)(t) = 0$

لان v_0 و $v_f = 0$

start from rest & come to rest

لو الرسمة بقت



الرسمه هنا على الارض

هيكون فى N + الحركه مش هتبدى من صفر لان اول بيكون وزن الجسم أكبر من القوة P و يبدأ حساب الحركه من الوقت الذى يصبح عنده P = W وزن الجسم

وبالتالى تكون فترة التكامل لا تبدأ من الصفر

$\int^x$ المطلوب الحساب عنده

وقت بداية الحركه

مثال $\boxed{P = 20t} \Rightarrow t = \frac{2(9.8)}{20}$

29/04/2013

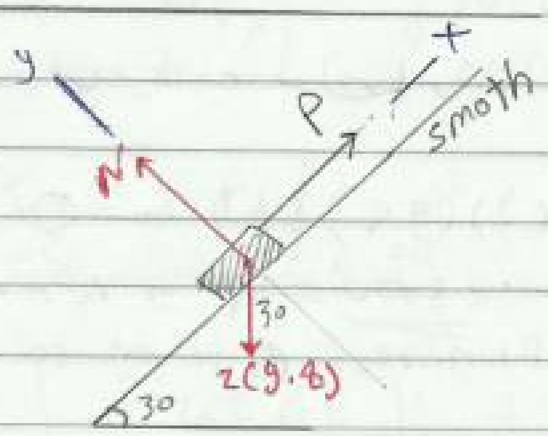
Impluse & momentum principle

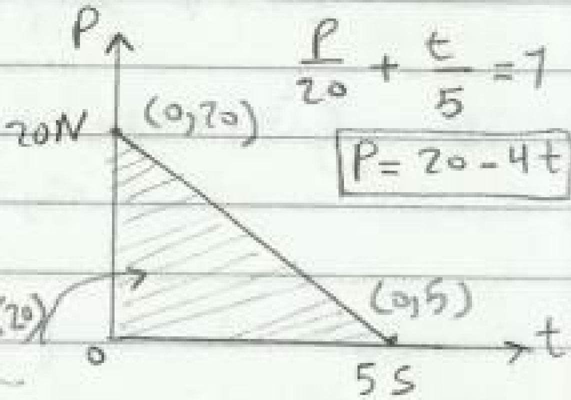
$$\boxed{I_x = \Delta P_x} \Rightarrow \int_{t_1}^{t_2} F_x \, dt = m(v_2)_x - m(v_1)_x$$

$$\boxed{I_y = \Delta P_y} \Rightarrow \int_{t_1}^{t_2} F_y \, dt = m(v_2)_y - m(v_1)_y$$

problem (13.138) page 816

mass = 2Kg $\qquad v_0 = 0$





$$\frac{P}{20} + \frac{t}{5} = 1$$

$$\boxed{P = 20 - 4t}$$

calalate : v_{5s} , time when the velocity $v = 0$

friction فيه لما يكون N بحسب *

$$\boxed{I_x = \Delta P_x} \qquad \int_0^5 (P - (2)(9.8)\sin 30)\, dt = 2v_{5s} - 2(0)$$

$$\underbrace{\int_0^5 P \, dt}_{50} - [2(9.8)\sin 30](5) = 2 v_{5s}$$

$v_{5s} = \checkmark$

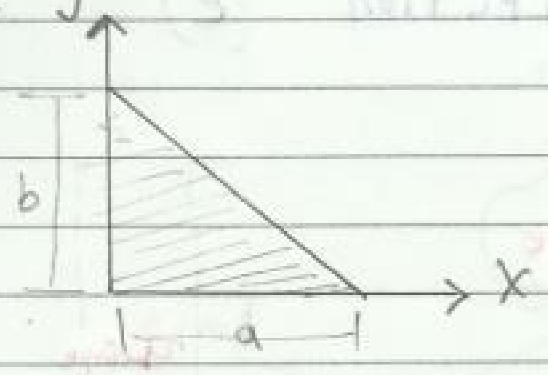
when the block come to stop $v_{final} = 0$
at time $t = ??$

$$\underbrace{\int_0^t P}_{=50} - (2(9.8)\sin 30)t = 2(\overset{zero}{v_{final}}) - 2(\overset{zero}{v_0})$$

$t = \checkmark$

تشابه الـ 5s P=zero

Note :-



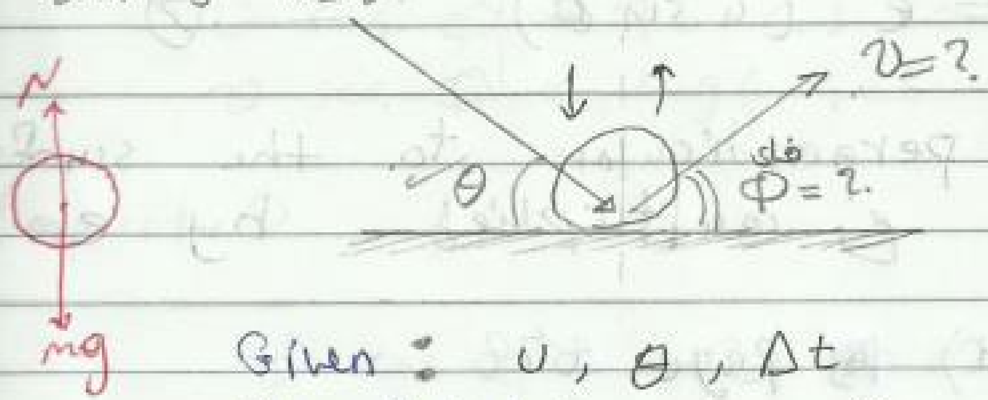
$$\frac{y}{b} + \frac{x}{a} = 1$$

$$\int y \, dx =$$

Area under curve.

Impact with a fixed surface :

u سرعة الصدم



Given : $u, \theta, \Delta t$

Requiried : v, ϕ

$\boxed{I_x = \Delta P_x}$

$$\int_0^{\Delta t} (0)\, dt = m(v \cos \phi) - m(u \cos \theta)$$

Force مفيش في اتجاه X

$$\boxed{\cancel{m}\, v \cos \phi = \cancel{m}\, u \cos \theta} \quad ①$$

Tangential component of velocity is constant

$\boxed{I_y = \Delta P_y}$

$$\boxed{\int_0^{\Delta t} (N - mg)\, dt = m(v \sin \phi) - m(u \sin \theta)}$$

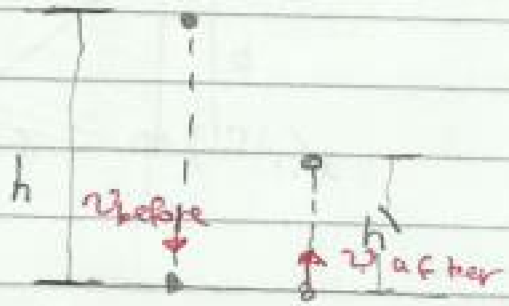
②

coefficient of restitution (e)

$$\boxed{v_{after} = e \; v_{before}}$$

تتوقف على نوع ال matrial

(معامل المرونة)



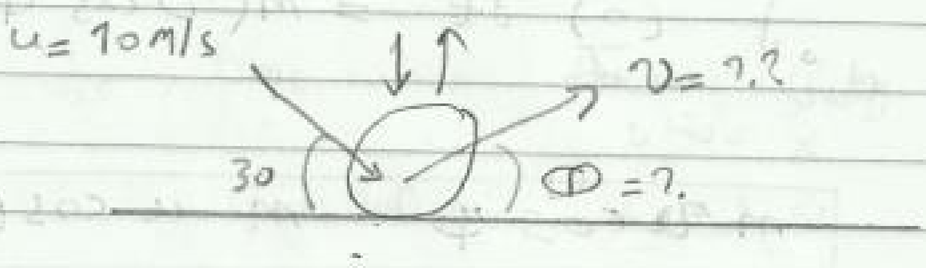
$$e \in [0,1]$$

$$\boxed{v \sin \phi = e \; (u \sin \theta)} \quad \text{—— (2)}$$

The velocity perpendicular to the surface is reversed & multiplied by e

Example (13.14) page 828

$e = 0.9$

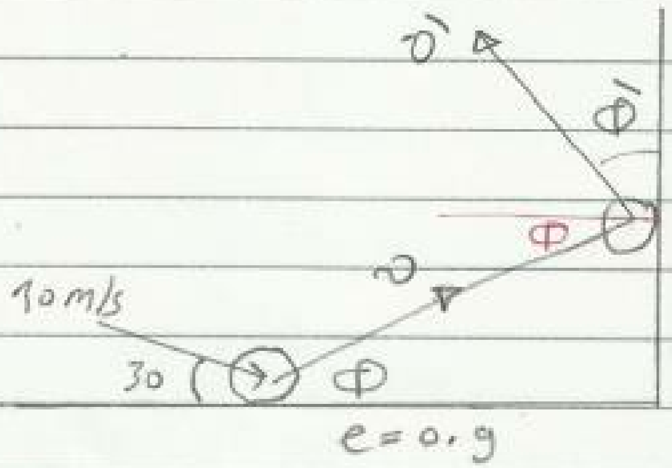


$$v \cos \phi = 10 \cos 30 \quad \text{—— (1)}$$

$$v \sin \phi = 0.9 \, (10 \sin 30) \quad \text{—— (2)}$$

$$\frac{(2)}{(1)} \qquad \tan \phi = 0.9 \tan 30$$

$$v = \checkmark$$



$e = 0.9$

Impact ①

$$\left.\begin{aligned} v\cos\phi &= 10\cos 30 \\ v\sin\phi &= 0.9(10\sin 30) \end{aligned}\right\} \begin{matrix} v \\ \phi \end{matrix}$$

Impact ②

$$\left.\begin{aligned} v'\cos\phi' &= v\sin\phi \\ v'\sin\phi' &= 0.9(v\sin\phi) \end{aligned}\right\} \begin{matrix} v' \\ \phi' \end{matrix}$$